MATH 1271: CALCULUS I
MIDTERM TEST I: ANSWERS TO THE SAMPLE EXAM

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Below are answers, not complete solutions. On the test, you will have to show all work.

(1) (1) $-1/10$; (2) 0; (3) 0; (4) $-\infty$; (5) $-\infty$; (6) 1; (7) does not exist.

(2) The roots are 0, one root on $(-2,-1)$, and one root on $(1,2)$ by IVT.

(3) (I have changed the problem from the original version to allow $x < 0$.) **Answer:** $b > 0$. **Solution:** The function is continuous at $x \neq 0$ or 1, because near other points, $f(x)$ is given as a composition of elementary functions. The one sided limits at $x = 1$ are always equal to 1, which is $f(1)$, therefore, the function is continuous at $x = 1$ for all b's. The limit of $f(x)$ at $x = 0$ is $0 = f(0)$, if $b > 0$; 1, if $b = 0$, and ∞, if $b < 0$. Since for $b = 0$, the function is not defined at $x = 0$ (0^0 is not defined), $f(x)$ may not be continuous at 0. It will not be continuous at $x = 0$ for $b < 0$, because of the infinite limit. Thus, the only values of b for which the function is continuous at $x = 0$ are $b > 0$.

(4) $f'(1) = 4$ and $y = 4x - 2$.

(5) **Solution:** $v(a) = s'(a) = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h} = \lim_{h\to 0} \frac{(a+h)^3-3(a+h)^2+3(a+h)-129-a^3+3a^2-3a+129)}{h} = \lim_{h\to 0} \frac{3a^2h+3ah^2+h^3-6ah-3h^2+3h}{h} = \lim_{h\to 0}(3a^2+3ah+h^2-6a-3h+3) = 3a^2-6a+3$. Set $v(a) = 0$ and get $3a^2-6a+3 = 0$, equivalent to $0 = a^2-2a+1 = (a-1)^2$. Thus, the **answer** is: at time $a = 1$.

(6) **Solution:** The rate of change of the area $A(t) = \pi r^2$ is its derivative $A'(t)$ with respect to time t, when $r = 50$ cm. By definition $A'(t) = \lim_{\Delta t\to 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta t\to 0} \frac{\pi(50+\Delta r)^2-\pi 50^2}{\Delta t} = \pi \lim_{\Delta t\to 0} \frac{2\cdot 50\Delta r+(\Delta r)^2}{\Delta t} = 100\pi \lim_{\Delta t\to 0} \frac{\Delta r}{\Delta t} + \pi \lim_{\Delta t\to 0} \frac{(\Delta r)^2}{\Delta t} = 100\pi r'(t) + \lim_{\Delta t\to 0} \frac{\Delta r}{\Delta t} \cdot \lim_{\Delta t\to 0} \Delta r = 100\pi r'(t) + r'(t)\cdot 0 = 100\pi \cdot 0.01 = \pi$ cm^2/min.

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