MATH 1271: CALCULUS I
MIDTERM TEST 3: ANSWERS AND SELECTED SOLUTIONS TO THE SAMPLE EXAM

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Below are answers, not complete solutions, except for Problems 5 and 6. On the test, you will have to show all work.

(1) Hint: Apply the Mean Value theorem to $f(x)$ on $[1, e]$.
(2) (1) 0; (2) -4/3; (3) 1; (4) 1.
(3) Max: $x = 0$; min: $x = \ln 3$; inflection point: $x = (\ln 3)/2$.
(4) 1/3. Note that you were asked to do the problem the "hard way," namely by computing the limit $\lim_{n\to\infty} \sum_{i=1}^{n} (i/n)^2/n$ directly.
(5) **Solution:** We need to optimize the difference of areas function $A(c) = \int_0^c \sin x dx - \int_0^c \cos x dx$ on the interval $[0, \pi/2]$. First, let us find the critical points. By FTC1 (or from an explicit computation of $A(c)$ below), $A'(c) = \sin c - \cos c$. Solving $A'(c) = 0$ on $[0, \pi/2]$, we get $c = \pi/4$, which is the only critical point. Now compare $A(\pi/4)$ to the values $A(0)$ and $A(\pi/2)$ at the endpoints.

$$A(c) = -\cos x]_0^c - [\sin x]_0^c = -\cos c + 1 - \sin c,$$

so $A(0) = A(\pi/2) = 0$ and $A(\pi/4) = 1 - \sqrt{2} < 0$. Thus, the minimum value is $1 - \sqrt{2}$, taken at $c = \pi/4$, and the maximum value is 0. Whence, the **answer:** (1) $c = \pi/4$; (2) 0.
(6) (1) Use substitution $u(x) = 1 + \sin x$. Then from computing du/dx, we get $du = \cos x dx$ and the integral is

$$\int du/u == \ln|u| + C = \ln|1 + \sin x| + C = \ln(1 + \sin x) + C,$$

where the last step is because $\sin x + 1 \geq 0$ for all x.

(2) Use $u = 3 - 2x$. Then $du = -2dx$, and the integral rewrites as

$$-\int \frac{du}{2\sqrt{u}} = -\sqrt{u} + C = -\sqrt{3 - 2x} + C.$$

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(3) Use $u = \log_3 x = \ln x/\ln 3$. Then $du = dx/(x \ln 3)$ and the integral becomes

$$\ln 3 \int u^3 du = \frac{\ln 3}{4} u^4 + C = \frac{\ln 3}{4} \log_3^4 x + C.$$