

Name: \_\_\_\_\_

**MATH 1271: CALCULUS I  
SAMPLE MIDTERM TEST 3**

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You have to show all work! A correct answer with incorrect or no work shown will not be counted.

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

**Problem 1.** Show that for  $f(x) = x + (\ln x)^2$ , the equation  $f'(x) = \frac{e}{e-1}$  has a solution  $x$  between 1 and  $e$ .

**Problem 2.** Find the following limits.

$$(1) \lim_{x \rightarrow 1^+} \frac{\sin\left(\frac{3\pi}{2}x\right) + 1}{\ln x};$$

$$(2) \lim_{x \rightarrow 0} \frac{\sin 2x - 2x}{x^3};$$

$$(3) \lim_{x \rightarrow 0} (\tan x)^{2 \sin x};$$

$$(4) \lim_{x \rightarrow \infty} x e^{1/x} - x.$$

**Problem 3.** Find the local maxima, the local minima, and the inflection points of the function  $f(x) = e^x - 3e^{-x} - 4x$ .

**Problem 4.** Use the definition of the integral as the limit of Riemann sums to evaluate

$$\int_0^1 x^2 dx.$$

[Hint: You may use the formula  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ .]

**Problem 5.** (1) On the interval  $[0, \pi/2]$ , find a point  $c$  such that the difference between the areas of the regions lying under the graphs of  $y = \sin x$  and  $y = \cos x$  from 0 to that point  $c$  has a minimal value.

(2) What is the maximal value of this difference?

**Problem 6.** Find the following indefinite integrals:

$$(1) \int \frac{\cos x}{1 + \sin x} dx;$$

$$(2) \int \frac{dx}{\sqrt{3 - 2x}};$$

$$(3) \int \frac{\log_3^3 x}{x} dx.$$