

**MATH 1272: CALCULUS II**  
**MIDTERM TEST II: A SAMPLE PROBLEM SET**

INSTRUCTOR: ALEX VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. Here is a list of trigonometric substitutions, which you may or may not need for the test:

Expression:  $\sqrt{a^2 - x^2}$

Substitution:  $x = a \sin \theta$ .

Expression:  $\sqrt{a^2 + x^2}$

Substitution:  $x = a \tan \theta$ .

Expression:  $\sqrt{x^2 - a^2}$

Substitution:  $x = a \sec \theta$ .

Good luck!

**Problem 1.** Find the length of the curve  $y = \sqrt{1 - x^2}$  between  $x = \sqrt{2}/2$  and  $x = 1$ .

**Problem 2.** Find the area of the surface obtained by rotating the following curve around the  $y$ -axis:

$$y = 4 - x^2, \quad 0 \leq x \leq 1.$$

**Problem 3.** Find the centroid of the region bounded by the curves

$$y = 3x^2, y = 0, x = 1.$$

**Problem 4.** Find the solution of the following initial value problem:

$$\frac{dy}{dx} = xe^{2y}, \quad y(0) = 1.$$

**Problem 5.** Brine flows at a constant rate of 2 L/min into a large tank that initially held 1000 L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 2 L/min. The concentration of salt in the brine entering the tank is 0.2 kg/L. Let  $y(t)$  be the amount of salt in the tank after  $t$  minutes. Find a differential equation and an initial condition that describe  $y$ . Find  $y(t)$ .

**Problem 6.** "Bolognium-37" has a half-life of 160 days. A sample has a mass of 200 grams.

- (1) Find the mass of the sample after 100 days.
- (2) After how many days will the mass be reduced to 5 grams?

**Problem 7.** Eliminate the parameter to find a Cartesian equation of the following parametric curve:

$$x = \cos^2 t + 1, y = 2 \sin t, \quad 0 \leq t \leq \pi.$$

Sketch the curve. Pick three different values of  $t$  and mark the corresponding points on the curve. Indicate with an arrow the direction in which the curve is traced as the parameter  $t$  increases.

**Problem 8.** Find the equation of the tangent to the curve at the point corresponding to the given value of the parameter:

$$x = t + 3, \quad y = \frac{1}{4}t^4 - t^2, \quad t = 2.$$

Find the points at which the tangent is horizontal. Find the second derivative  $d^2y/dx^2$  at  $t = 2$ .

**Problem 9.** Find the length of the curve

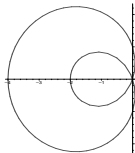
$$x = \cos \theta + \theta \sin \theta, \quad y = \sin \theta - \theta \cos \theta, \quad 0 \leq \theta \leq \pi.$$

**Problem 10.** Sketch the curve with the polar equation

$$r = 3 \cos \theta, \quad 0 \leq \theta \leq \frac{3\pi}{2}.$$

Set up an integral for the length of the curve in polar coordinates and evaluate the integral.

**Problem 11.** Set up an integral giving the area of the inner loop of the curve  $r = 1 - 3 \cos \theta$ :



**Problem 12.** Find the vertices, foci, and asymptotes of the hyperbola  $x^2 - y^2 = 1$ .

**Problem 13.** Find the limit of the sequence  $\{\sqrt{n+2} - \sqrt{n}\}$ , if it exists.