MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM TEST IV

INSTRUCTOR: SASHA VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!

Problem 1. Let $V = \{(x, y, z) \in \mathbb{R}^3 : x + y - z = 0\}$. Find a basis of V.

Problem 2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about the line $y = x$. Find the matrix A of the linear transformation T and the eigenspaces of A.

Problem 3.
Determine whether the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$$

is diagonalizable. If it is, find a matrix S that diagonalizes A and determine $S^{-1}AS$.

Problem 4. Solve the IVP:

$$\begin{aligned} \mathbf{x}' &= \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \mathbf{x}, \\ \mathbf{x}(\mathbf{0}) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \end{aligned}$$

Problem 5. Determine the general solution to the system $\mathbf{x}' = A\mathbf{x}$ for

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & -1 & 3 \\ 2 & -1 & 3 \end{bmatrix}.$$

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