# MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
# SAMPLE MIDTERM TEST II

INSTRUCTOR: ALEX VORONOV

You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

**Problem 1.** Use Gaussian elimination to transform the augmented matrix of the following system into the echelon form. Use it to find the solutions, if there exist any.

$$\begin{aligned} x + y - 2z &= 0, \\ 3x + 5y - 2z &= 8. \end{aligned}$$

**Answer**:

$$\begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

$$\begin{aligned} x &= 4t - 4 \\ y &= 4 - 2t \\ z &= t \end{aligned}$$

**Problem 2.** (1) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

**Answer**:

$$A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & -1 & 1 \\ -1 & 2 & -2 \end{bmatrix}.$$

(2) Use the inverse of $A$ to solve the system (another way of solving it will not be counted)

$$\begin{aligned} 2y + z &= 1, \\ x + z &= 0, \\ x - y &= -1. \end{aligned}$$

**Answer**: $(-1, 0, 1)$.

---

*Date*: October 27, 2009.

**Problem 3.** Use Cramer's rule to determine the unique solution to the system $A\mathbf{x} = \mathbf{b}$ for the following matrix and vector:

$$A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 5 \\ 2 & 3 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix}.$$

**Answer**: $x_1 = 1/4, \qquad x_2 = 1/16, \qquad x_3 = 21/16.$

**Problem 4.** Determine whether or not the set

$$S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$$

is a subspace of $\mathbb{R}^2$. Justify your answer.

**Answer**: No.

**Problem 5.** An object of mass 2 kg, resting on a table next to a wall, is attached to the wall by a spring. A force of 8 N is applied to the mass, stretching the spring and moving the mass 1/2 m from its equilibrium position. The object is then released. Suppose the resistance to the motion is numerically equal to 8 times the instantaneous velocity.

(1) Set up an IVP governing the motion of the mass.
**Answer**:
$$2x'' + 8x' + 16x = 0, \qquad x(0) = 1/2, \quad x'(0) = 0.$$
(2) Determine the position of the mass at any time $t$.
**Answer**:
$$x = e^{-2t}(\cos 2t + \sin 2t)/2.$$
(3) At what time does the mass first pass through the equilibrium position and heading away from the wall?
**Answer**: $t = 7\pi/8$.