

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL
EQUATIONS
SAMPLE MIDTERM TEST III**

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You may not use calculators, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Solve the initial value problem

$$y'' + 2y' + 2y = 4x^2 + e^{-x} \cos x, \quad y(0) = -1, \quad y'(0) = 1.$$

Answer:

$$y = e^{-x}(-3 \cos x + 2 \sin x) + 2x^2 - 4x + 2 + (xe^{-x} \sin x)/2.$$

Problem 2. (1) Determine whether the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$$

is diagonalizable. If it is, find a matrix S that diagonalizes A and determine $S^{-1}AS$.

Answer:

$$S = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}, \quad D = S^{-1}AS = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}.$$

(2) Solve the IVP:

$$\begin{aligned} \mathbf{x}' &= A\mathbf{x}, \\ \mathbf{x}(0) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \end{aligned}$$

Answer:

$$\mathbf{x} = \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix}.$$

Problem 3. Suppose we have two tanks as in the mixing problem in Figure 7.1.3 on p. 397 of the text, except that there is no inflow from or outflow to the outside, and the exchange rate between the two tanks is 2 L/min each way. Suppose that tank 1 contains 6 L of solution and tank 2 contains 12 L of solution, and that initially tank 1 contains 5g of chemical and tank 2 contains 25 g of chemical.

(1) Determine the amount of chemical in each tank at time t .

Answer:

$$\begin{aligned} x &= 10 - 5e^{-t/2}, \\ y &= 20 + 5e^{-t/2}. \end{aligned}$$

(2) Eventually, what will be the amount of chemical in each tank?

Answer: $x = 10$ and $y = 20$.

Problem 4. Given $\mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$, use the Laplace transform to solve the following IVP:

$$y'' + 4y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = -2.$$

Answer: $Y = -\frac{2}{s^2+4s+4}$ and $y = -2te^{-2t}$.