

**MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL
EQUATIONS
SAMPLE MIDTERM TEST I**

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You may not use notes, books, etc. Only the exam paper, a pencil or pen, and a basic or scientific calculator may be kept on your desk during the test.

Good luck!

Problem 1. Find all solutions to the following differential equations:

(1) $y^2 y' = \cos 6x$, $y(0) = 2$;

(2) $y' = x + e^{-x^2/2} - xy$.

Problem 2. The radioactive isotope Indium-111 is often used for diagnosis and imaging in nuclear medicine. Its half life is 2.8 days. What was the initial mass of the isotope before decay, if the mass in 2 weeks was 6g?

Problem 3. A commercial fishery is estimated to have carrying capacity of 10 thousand pounds of certain kind of fish. Suppose the annual growth rate of the total fish population P , measured in thousand pounds, is governed by the logistic equation

$$\frac{dP}{dt} = \left(1 - \frac{P}{10}\right)P,$$

and initially there is a total of 2 thousand lbs of fish. What is the fish population after 1 year?

Problem 4. (1) Write the augmented coefficient matrix of the following linear system and transform it to echelon form:

$$2x_1 + 4x_2 - x_3 - 2x_4 + 2x_5 = 6,$$

$$x_1 + 3x_2 + 2x_3 - 7x_4 + 3x_5 = 9,$$

$$5x_1 + 8x_2 - 7x_3 + 6x_4 + x_5 = 4.$$

(2) Use the echelon matrix in (1) to solve the system by back substitution.

Solution: (1)

$$\begin{bmatrix} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix}$$

There are several ways to proceed from now. You may start with swapping, $\text{SWAP}(R_1, R_2)$, the first two rows:

$$\begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 2 & 4 & -1 & -2 & 2 & 6 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix},$$

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then subtracting $2R_1$ from R_2 and $5R_1$ from R_3 to eliminate the entries in the first column under the 1. You will get this:

$$\begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & -2 & -5 & 12 & -4 & -12 \\ 0 & -7 & -17 & 41 & -14 & -41 \end{bmatrix}.$$

Then do $-7R_2/2 + R_3$ to eliminate the first nonzero entry in the third row R_3 and get this:

$$\begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & -2 & -5 & 12 & -4 & -12 \\ 0 & 0 & 1/2 & -1 & 0 & 1 \end{bmatrix}.$$

This is an echelon matrix, and we are done.

(2) Write out the linear equations from the echelon matrix you have obtained in Part (1). Here is what I got. Note that your answer to (1) could be different and thereby you could have gotten a different linear system.

$$\begin{aligned} x_1 + 3x_2 + 2x_3 - 7x_4 + 3x_5 &= 9, \\ -2x_2 - 5x_3 + 12x_4 - 4x_5 &= -12, \\ \frac{1}{2}x_3 - x_4 &= 1. \end{aligned}$$

The leading variables are x_1 , x_2 , and x_3 . Thus, x_4 and x_5 must be set free. Say:

$$\begin{aligned} x_4 &= s, \\ x_5 &= t. \end{aligned}$$

Then from the three equations above, by back substitution, working upward, we get the following.

$$\begin{aligned} x_3 &= 2 + 2s, \\ x_2 &= 1 + s - 2t, \\ x_1 &= 2 + 3t. \end{aligned}$$