

Math 4606. Fall 2006.

**EXAM 1**

ID: ..... Date: .....

Name: .....

1. (20 points) Let  $X$  and  $Y$  be two non-empty sets and let  $f$  be a one-to-one function from  $X$  to  $Y$ . Let  $A$  be a subset of  $X$ . Show that  $f(X \setminus A)$  is a subset of  $Y \setminus f(A)$ .

2. (20 points) Let  $f$  be a function from  $\mathbb{R}^2$  to  $\mathbb{R}$  given by

$$f(x, y) = \begin{cases} \frac{2xy}{x^2+5y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Does the limit  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exist? Why? Find the limit if it does.

3. (20 points) Let  $f, g$  and  $h$  be three real-valued functions on  $\mathbb{R}^n$  satisfying

$$g(x) \leq f(x) \leq h(x) \text{ for all } x \in \mathbb{R}^n.$$

Let  $a \in \mathbb{R}^n$  and  $L \in \mathbb{R}$  and suppose that

$$\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L.$$

Prove that  $\lim_{x \rightarrow a} f(x) = L$ .

4. (20 points) Show that the set

$$S = \{(x, y) \in \mathbb{R}^2 : xy > 5 \text{ and } y + x^2 + 3x < 13\}$$

is an open set in  $\mathbb{R}^2$ .

5. (20 points) Find the limit

$$\lim_{k \rightarrow \infty} \frac{-3k^3 + 8k^2 - 7k + 11}{4k^3 - k^2 + 5}.$$