

Math 4606. Fall 2006.

**EXAM 2**

ID: ..... Date: .....

Name: .....

1. (25 points) Let  $A$  and  $B$  be two compact subsets of  $\mathbb{R}^n$ . Define the distance between  $A$  and  $B$  by

$$d(A, B) = \inf\{|x - y| : x \in A, y \in B\}.$$

Show that if  $A \cap B = \emptyset$  then  $d(A, B) > 0$ .

2. (25 points) Show that  $f(x) = 2\sqrt{x} - 3\cos x + \ln(x^2 + 1)$  is uniformly continuous on  $(1, \infty)$ .

3. (25 points) Let  $S$  be a connected set in  $\mathbb{R}^3$  containing two points  $(1, 2, 0)$  and  $(-1, 3, 6)$ . Show that  $S$  contains at least one point on the plane defined by the equation  $3x - y + 2z - 5 = 0$ .

4. (25 points) Find the limit

$$\lim_{x \rightarrow 0} \frac{x^2 + 2\cos x - 2}{x^4}.$$