

Practice problems for the Final, part 3

Note: Practice problems for the Final Exam, part 1 and part 2 are the same as Practice problems for Midterm 1 and Midterm 2.

1. Calculate Fourier Series for the function $f(x)$, defined on $[-2, 2]$, where

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0, \\ 2, & 0 < x \leq 2. \end{cases}$$

2. Calculate Fourier Series for the function $f(x)$, defined on $[-5, 5]$, where

$$f(x) = 3H(x - 2).$$

3. Calculate Fourier Series for the function, $f(x)$, defined as follows:

- (a) $x \in [-4, 4]$, and

$$f(x) = 5.$$

- (b) $x \in [-\pi, \pi]$, and

$$f(x) = 21 + 2 \sin 5x + 8 \cos 2x.$$

- (c) $x \in [-\pi, \pi]$, and

$$f(x) = \sum_{n=1}^8 c_n \sin nx, \quad \text{with } c_n = 1/n.$$

- (d) $x \in [-3, 3]$, and

$$f(x) = -4 + \sum_{n=1}^6 c_n (\sin(\pi nx/3) + 7 \cos(\pi nx/3)), \quad \text{with } c_n = (-1)^n.$$

4. (a) Let $f(x) = x + x^3$ for $x \in [0, \pi]$. What coefficients of the Fourier Series of f are zero? Which ones are non-zero? Why?

- (b) Let $g(x) = \cos(x^5) + \sin(x^2)$. What coefficients of the Fourier Series of g are zero? Which ones are non-zero? Why?