5. Let $f(x) = 2x + x^4$ for $x \in [0, 5]$.

   (a) Write down the function $G(x)$, which is the odd continuation for $f(x)$. Specify what terms will be zero and non-zero in the Fourier expansion for $G(x)$.

   (b) Write down the function $V(x)$, which is the even continuation for $f(x)$. Specify what terms will be zero and non-zero in the Fourier expansion for $V(x)$.

6. Suppose $f(x)$ is defined for $x \in [0, 7]$, and $f(x) = 2e^{-4x}$. Another function, $F(x)$, is given by the following:

$$F(x) = \sum_{n=0}^{\infty} a_n \cos(\pi n x/7),$$

   where

$$a_n = \frac{2}{7}\int_0^7 2e^{-4x} \cos\left(\frac{\pi n x}{7}\right) dx.$$

   What is the value of $F(3)$? What is the value of $F(-2)$?

7. Let us supposed that both ends of a string of length 25 cm are attached to fixed points at hight 0. Initially, the string is at rest, and has the shape $4\sin(2\pi x/25)$, where x is the horizontal coordinate along the string, with zero at the left end. The speed of wave propagation along the string is $3\,cm/sec$. Write down the complete initial and boundary value problem for the shape of the string.

8. Let us suppose that the following boundary value problem is given:

$$\frac{\partial^2 y}{\partial t^2} = 50\frac{\partial^2 y}{\partial x^2}, \quad x \in [0, 100], \tag{1}$$

$$y(0,t) = y(100,t) = 0, \tag{2}$$

$$y(x,0) = x^2(100 - x), \tag{3}$$

$$\frac{\partial y(x,0)}{\partial t} = \begin{cases} x, & 0 \le x \le 25, \\ 1/3(100 - x), & 25 \le x \le 100. \end{cases} \tag{4}$$

   What is the speed of wave propagation along the string? What is the initial displacement of the string at point $x = 20$? What is the initial velocity of the string at point $x = 50$? At what point of the string is the initial velocity the largest?