

9. Let us suppose that the following boundary value problem is given:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}, \quad x \in [0, 2], \quad (5)$$

$$y(0, t) = y(\pi, t) = 0, \quad (6)$$

$$y(x, 0) = 0, \quad (7)$$

$$\frac{\partial y(x, 0)}{\partial t} = g(x). \quad (8)$$

Suppose that

$$\int_0^2 g(x) \sin\left(\frac{\pi n x}{2}\right) dx = \frac{1}{n^3}.$$

Find $y(x, t)$.

10. Let us suppose that the following boundary value problem is given:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}, \quad x \in [0, \pi], \quad (9)$$

$$y(0, t) = y(\pi, t) = 0, \quad (10)$$

$$y(x, 0) = 22 \sin 2x + 8 \sin 6x, \quad (11)$$

$$\frac{\partial y(x, 0)}{\partial t} = 0. \quad (12)$$

Find $y(x, t)$ in a closed form (containing no integrals). You will not need to evaluate any integrals.