

Math 4606. Spring 2007
Solutions to Homework 6

Problem 2.1.4. $h'(0) = \lim_{x \rightarrow 0} h(x)/x = 0$, since $|h(x)| \leq x^2$ for all x .

Problem 2.1.5. For $h > 0$, $(f(a+h) - f(a))/h = f'(c)$ for some $c \in (a, a+h)$.

As $h \rightarrow 0$, $c \rightarrow 0$ also, and so $f'(c) \rightarrow L$.

Problem 2.2.1. (a) See the answer in the back of the text.

(b) $\nabla f(x, y) = (4e^{4x-y^2}, -2ye^{4x-y^2})$; $\nabla f(1, -2) = (4, 4)$; $\partial_{(3/5, 4/5)} f(1, -2) = \frac{3}{5} \cdot 4 + \frac{4}{5} \cdot 4 = \frac{28}{5}$.

(c) $\nabla f(x, y) = (-11y-14, 11x-12)/(7x+3y)^2$; $\nabla f(1, -2) = (8, -1)$; $\partial_{(3/5, 4/5)} f(1, -2) = \frac{3}{5} \cdot 8 - \frac{4}{5} \cdot 1 = 4$.

Problem 2.2.3. $dw = \frac{2xy^{3/2}z}{z+1} dx + \frac{3x^2y^{1/2}z}{2(z+1)} dy + \frac{x^2y^{3/2}}{(z+1)^2} dz$, so $dw|_{(5, 4, 1)} = 40dx + \frac{75}{2}dy + 50dz$.

(a) $0 = 40(.03) + \frac{75}{2}(-.08) + 50dz \implies dz = (-1.2 + 3)/50 = .036$.

(b) The coefficient of dz is largest.

Problem 2.2.5. $\partial_x u = -y^2/(xy - y + 2x)^2$ and $\partial_y u = 2x^2/(xy - y + 2x)^2$; the result follows.

Problem 2.2.6. Since $\partial_j(|\mathbf{x}|^{-1}) = -x_j|\mathbf{x}|^{-3}$, we have $df_i = |\mathbf{x}|^{-1}dx_i - \sum_{j=1}^n x_i x_j |\mathbf{x}|^{-3} dx_j$ and hence

$$\begin{aligned} \sum_i x_i df_i &= \sum_i x_i |\mathbf{x}|^{-1} dx_i - \sum_i \sum_j x_i^2 x_j |\mathbf{x}|^{-3} dx_j \\ &= \sum_i x_i |\mathbf{x}|^{-1} dx_i - \sum_j x_j |\mathbf{x}|^{-1} dx_j = 0. \end{aligned}$$