Math 4606. Spring 2007

Solutions to Homework 7

Problem 2.2.2. (a) See the answer in the back of the text.
(b) $df(x, y, z) = (x + z^2)^{-1}dx + 3y^2dy + 2z(x + z^2)^{-1}dx$; $df(1, 1, 0) = dx + 3dy$; $f(1.1, 1.2, -0.1) - f(1, 1, 0) \approx 0.1 + 3(0.2) = 0.7$.

Problem 2.2.8. Assume $n = 2$ for simplicity; the general case follows by an elaboration of the argument as in the proof of Theorem 2.19. Suppose $|\partial_1 f| \leq C$ and $|\partial_2 f| \leq C$ on S. Given $\mathbf{a} \in S$, let $r > 0$ be small enough so that $B(r, \mathbf{a}) \subset S$. If $|\mathbf{h}| < r$, by the Mean Value Theorem we have

$$\begin{aligned} |f(\mathbf{a} + \mathbf{h}) - f(\mathbf{a})| &\leq |f(a_1 + h_1, a_2 + h_2) - f(a_1, a_2 + h_2)| \\ &\quad + |f(a_1, a_2 + h_2) - f(a_1, a_2)| \\ &\leq C(|h_1| + |h_2|), \end{aligned}$$

which implies the continuity of f at $\mathbf{a}$.

Problem 2.3.2. (a) and (c) See the answer in the back of the text.
(b) $\partial_x w = e^{x-3y}f_1 + 2xf + 2/(x^2 + 1)$, $\partial_y w = -3e^{x-3y}f_1 + 2y^3 f_3/\sqrt{y^4 + 4}$.

Problem 2.3.5. Both formulas for the tangent plane at $(a, b, f(a, b))$ amount to $z - f(a, b) = A(x - a) + B(y - b)$, where $A = \partial_x f(a, b)$ and $B = \partial_y f(a, b)$.

Problem 2.3.6. These are all similar; I will just do (d). If $F(x, y, z) = xyz^2 - \log(z - 1)$, we have $\partial_x F = yz^2$, $\partial_y F = xz^2$, and $\partial_z F = 2xyz - (z - 1)^{-1}$, so $\nabla F(-2, -1, 2) = (-4, -8, 7)$. Hence the tangent plane is given by $-4(x + 2) - 8(y + 1) + 7(z - 2) = 0$, or $7z = 4x + 8y + 30$.

Problem 2.4.2. (a) If S is convex, we can apply Theorem 2.39 to get $f(\mathbf{b}) - f(\mathbf{a}) = \partial_1 f(\mathbf{c})(b_1 - a_1) = 0$.
(b) An example with $n = 2$: Let S be the square $(-1, 1) \times (-1, 1)$ with the segment $[0, 1)$ on the y axis removed. Define f on S by $f(x, y) = 0$ if $y \leq 0$, $f(x, y) = -y^2$ if $y > 0$ and $x < 0$, and $f(x, y) = y^2$ if $y > 0$ and $x > 0$. Then $\partial_x f = 0$ on S, but $f(-x, y) \neq f(x, y)$ for $y > 0$.