SOLUTIONS

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MATH 4606: ADVANCED CALCULUS
MIDTERM EXAM I

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!

Problem 1. Show that

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y - 3y^3}{x^2 + y^2} = 0.$$

First, estimate:

$$\frac{|y|\,|2x^2 - 3y^2|}{x^2+y^2} \le 3|y|, \text{ because}$$

(triangle inequality)

$$|2x^2 - 3y^2| \le 2x^2 + 3y^2 \le 3(x^2+y^2)$$

($x^2 \ge 0$)

Given $\varepsilon > 0$, take $\delta = \frac{\varepsilon}{3}$. Then for each (x,y) : $0 < |(x,y)| < \delta$, we'll have

$$\left|\frac{2x^2y - 3y^3}{x^2+y^2}\right| \le 3|y| \le 3|(x,y)| < 3\delta = \varepsilon.$$

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