Problem 3. Recall that a point $\mathbf{a} \in \mathbb{R}^n$ is called an *accumulation point* of a set $S \subset \mathbb{R}^n$, if every open ball around $\mathbf{a}$ contains infinitely many points of S. Show that every infinite bounded set in $\mathbb{R}^n$ has an accumulation point.

Let S be an infinite bounded set in $\mathbb{R}^4$. Take a ~~sequence~~ $\{\vec{X}_k\}$ of (pairwise) distinct points S. We can do that, because S is infinite.

Then $\{\bar{X}_k\}$ is a bounded sequence. Therefore, by a theorem, it has a convergent subsequen $\{\bar{X}_{i_k}\}$. Let $\vec{a} \in \mathbb{R}^n$ be its limit.

It's an accumulation point, because by definition of a limit, every open ball around $\vec{a}$ contains all terms of $\{X_{i_k}\}$ from ~~some~~ $k = K$ and on, i.e., infinitely many terms of $\{\bar{X}_k\}$, and, by construction, ∞ many points of S.