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MATH 4606: ADVANCED CALCULUS  
SAMPLE MIDTERM EXAM II

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. You must show all work.

Good luck!

**Problem 1.** Suppose  $f: [0, 1] \rightarrow \mathbb{R}$  is a continuous function such that  $f(0) = 1$  and  $f(1) = 0$ . Show that there exists  $c \in [0, 1]$  such that  $f(c) = \sqrt{c}$ .

Consider  $g(x) = f(x) - \sqrt{x}$ . It's a continuous function from  $(0, 1]$  to  $\mathbb{R}$ .  $g(0) = 1 - 0 = 1$ ,  $g(1) = 0 - 1 = -1$ . Thus, by the Intermediate Value Theorem,  $\exists c \in (0, 1]$  such that  $g(c) = 0$ . This means  $f(c) - \sqrt{c} = 0$  or  $f(c) = \sqrt{c}$ .