Problem 2. Show that if $f : S \to \mathbb{R}$ and $g : S \to \mathbb{R}$ are functions uniformly continuous on some domain $S \subset \mathbb{R}^n$, then $f - 2g$ is uniformly continuous on S.

Given $\varepsilon > 0$, take $\delta_1 > 0$: $\forall \vec{x}, \vec{y}$ ~~xx~~ $\in S$
$|\vec{x} - \vec{y}| < \delta_1 \Rightarrow |f(\vec{x}) - f(\vec{y})| < \frac{\varepsilon}{2}$ and

$\delta_2 > 0$: $\forall \vec{x}, \vec{y} \in S$ $\quad |\vec{x} - \vec{y}| < \delta_2 \Rightarrow |g(\vec{x}) - g(\vec{y})| < \frac{\varepsilon}{4}$

Let $\delta = \min(\delta_1, \delta_2)$. Then $\forall \vec{x}, \vec{y} \in S$ such that

$|\vec{x} - \vec{y}| < \delta$ we'll have

$$|f(\vec{x}) - 2g(\vec{x}) - f(\vec{y}) + 2g(\vec{y})| \le$$

$$\le |f(\vec{x}) - f(\vec{y})| + 2|g(\vec{x}) - g(\vec{y})| < \frac{\varepsilon}{2} + 2\frac{\varepsilon}{4} = \varepsilon.$$