

Problem 3. Show that if a differentiable function $f(x)$ on an interval (a, b) has a positive derivative, $f'(x) > 0$, for all $x \in (a, b)$, except for two points at which $f'(x) = 0$, then the function $f(x)$ is strictly increasing on (a, b) .

~~Let x_1, x_2 be the points on (a, b) at which $f'(x) = 0$. Then $\forall x, y: a < x < y < b$,~~

~~let~~

let x_1, x_2 be the points on (a, b) at which $f'(x) = 0$.
we'll show that f is strictly increasing on $(a, x_1]$, $[x_1, x_2]$, and $[x_2, b)$, which will imply $f(x)$ is strictly increasing on (a, b) .

$\forall x, y: a < x < y \leq x_1$, we have

$$f(y) - f(x) = f'(c)(y-x) \text{ for some } c \in (x, y) \text{ by MVT. Then } f'(c) > 0 \text{ and } y-x > 0$$

so $f(y) - f(x) > 0$. Similarly, we'll see that

Thus, f is increasing on $(a, x_1]$.
The other 2 intervals are done in the same way.