Problem 4. Find the extreme values of the function $f(x, y, z) = 3x^2 + y^2 + 2z^2$ on the unit **ball** $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$.

In the interior of the ball look for critical pts of f:

$\nabla f = (6x, 2y, 4z) = 0$ at $(x, y, z) = 0$, at which $f(\vec{x}) = 0$.

On the boundary, use Lagrange's method:

$\nabla f = \lambda \nabla G$, where $G = x^2 + y^2 + z^2 - 1$

$(6x, 2y, 4z) = \lambda(2x, 2y, 2z)$

$\begin{cases} 3x = \lambda x \\ y = \lambda y \\ 2z = \lambda z \end{cases} \Leftrightarrow$ $x = 0$ or $\lambda = 3$

$y = 0$ or $\lambda = 1$ $\quad$ $y = z = 0$, $x = \pm 1$

$z = 0, y = \pm 1$

~~z = 0~~ or ~~x = 0~~

$z = \pm 1$

$f(\pm 1, 0, 0) = 3$, $f(0, \pm 1, 0) = 1$, $f(0, 0, \pm 1) = 2$

Thus, the min value is 0 at $(0, 0, 0)$ and the max one is 3 at $(\pm 1, 0, 0)$.

Why not check up all your work?