Math 8202 **Homework 3**

Date due: February 15, 2010

Hand in only the starred questions. One of the other problems may show up on the upcoming quiz on February 22.

If you want me to do in class any of the questions I have given you, do let me know (on or after the due date for the assignment). For instance, if you want me to do on February 15 (22, respectively) or later any of the questions below (not, respectively) assigned to be handed in, please tell me.

Section 4.3: 2, 4, 5, 6*, 9, 10, 11, 13, 25, 29, 30, 31, 32, 34 (I list a lot of questions, and I expect that it will be appropriate for you to skim over many of them, simply looking to make sure you can do them.)

D. Let G be an infinite group containing a nontrivial element x having only finitely many conjugates. Prove that G is not simple.

E. Let $a = (1\ 2\ 3\ 4) \in S_4$. Describe the centralizer $C_{S_4}(a)$. (Determine its structure and its order.)

F. Show that when $a = (4\ 5) \in S_5$, the subgroup $C_{S_5}(a)$ consists of $S_3 \cup S_3 a$, where S_3 denotes the symmetric group on three symbols as a subgroup of S_5 permuting the symbols $\{1, 2, 3\}$.

G. (related to Exercise 4.3.10) Consider the permutation $a = (1\ 2\ 3\ 4\ 5)$.

(a) Show that a has 24 conjugates in S_5.

(b) Show that a has only 12 conjugates in A_5. [Hint: compute the index of $C_{A_5}(a)$ in A_5.]

(c) Show that a is conjugate in A_5 to $(5\ 4\ 3\ 2\ 1)$.

(d) Show that a is not conjugate in A_5 to $(1\ 3\ 5\ 2\ 4)$.

H. Let $a = (1\ 2\ 3\ 4)(5\ 6\ 7) \in S_7$.

(a) Find a permutation g of the symbols $\{1, 2, 3, 4, 5, 6, 7, 8\}$ so that

$$gag^{-1} = (2\ 1\ 6\ 5)(3\ 8\ 7).$$

Express g as a product of disjoint cycles.

(b) Calculate the number of conjugates of a in S_7. Calculate the number of conjugates of a in S_8.

(c) Show that the only elements of S_7 which commute with a are the powers of a.

I*. (Graduate Algebra Exam, Fall 2002) (18%)

(a) (4%) Calculate the numbers of conjugates of each of the elements $(1\ 2\ 3\ 4\ 5)$ and $(1\ 2\ 3)(4\ 5\ 6)$ in the symmetric group S_6. [We use cycle notation for permutations, writing them as a disjoint union of cycles.]

(b) (7%) Calculate the numbers of conjugates of each of the elements $(1\ 2\ 3\ 4\ 5)$ and $(1\ 2\ 3)(4\ 5\ 6)$ in the alternating group A_6.

(c) (7%) Show that $(1\ 2\ 3\ 4\ 5)$ and $(1\ 3\ 5\ 2\ 4)$ are not conjugate in A_6.

Section 4.4: 4, 5*, 8, 11*, 12, 13*.