Math 8202 **Homework 5**

Date due: March 1, 2010

Hand in only the starred questions. One of the other problems may show up on the quiz on March 8, the International Women's Day. :-)
If you want me to do in class any of the questions I have given you, do let me know (on or after the due date for the assignment).

Section 5.1: 1, 2, 4*, 5, 6, 18.

Section 5.4: 2, 4, 7*, 10, 11, 13, 15, 17, 19.

N*. Show that every group of order 1001 is cyclic.

O. Let G be the group of *all* isometries of the cube, and let H be the subgroup consisting of rotations which preserve the cube. Let -1 denote the element of G which is the transformation of $\mathbb{R}^3$ given by multiplication by -1.

(1) Show that $G = H \times \langle -1 \rangle$. [For subgroups H and K of a group G, we write $G = H \times K$ to emphasize that G is the internal direct product of H and K, i.e., $G = HK$, the subgroups are normal and intersect trivially.]
(2) Show that if $g \in G$ is any element of order 2 other than -1, then $G \neq H\langle -1 \rangle$. [To do this you may need to prove that the center of H is $\{1\}$. Either use the isomorphism with S_4 or note that if you conjugate one rotation by another rotation you get rotation about an axis obtained by applying the second rotation to the axis of the first.]

P*.

(1) Let G be the group of *all* isometries of the tetrahedron, and let H be the subgroup consisting of rotations. Determine whether or not $G = H \times K$ for some subgroup K of G.
(2) Let G be the group of *all* isometries of the icosahedron, and let H be the subgroup consisting of rotations. Determine whether or not $G = H \times K$ for some subgroup K of G.

Q. Show that the group $\mathrm{Aff}(V) = \{\mathbf{x} \mapsto A\mathbf{x} + \mathbf{b} \mid \mathbf{x} \in V, A \in \mathrm{GL}(V), \mathbf{b} \in V\}$ of affine transformations of a vector space V is a semidirect product $V \rtimes \mathrm{GL}(V)$.

R*. Show that the group Z_4 gives an example of an extension $Z_2 \trianglelefteq Z_4$ which is not isomorphic to a semidirect product $Z_2 \rtimes Z_2$.