Math 8202 **Homework 6**

Date due: March 8, 2010. There will be a quiz on this day

Hand in only the starred questions. One of the other problems may show up on the quiz.

If you want me to do in class any of the questions I have given you, do let me know (on or after the due date for the assignment).

S. Let B be the matrix of a bilinear form g with respect to a basis.

(1) Show that g is symmetric, iff $B^T = B$.
(2) Show that g is alternating, iff $B^T = B$ and has diagonal entries equal to zero.
(3) Show that g is Hermitian, iff $B^T = \bar{B}$.

T. Show that every linear map $\alpha : V \to V^*$ defines a bilinear form g so that the linear map corresponding to g is α: $\alpha(v) = g(-, v)$.

U. Show that if α is a symmetric linear operator, then eigenvectors with distinct eigenvalues are orthogonal.

V*. (Spring 1994, no. 6) Let A be a real $m \times n$ matrix. Regarding A as the matrix of a linear map $\mathbb{R}^n \to \mathbb{R}^m$, let V be a subspace of $\mathbb{R}^n$ so that $\mathbb{R}^n = V \oplus \ker A$. Show that the bilinear form $\langle , \rangle$ defined on V by

$$\langle u, v \rangle = u^T A^T A v \qquad \text{for all } u, v \in V$$

is nonsingular (a.k.a. nondegenerate). Hence show that the matrices A and $A^T A$ have the same rank.

W*. (Fall 2001) Let $(,)$ be the bilinear form on $\mathbb{R}^3$ specified on column vectors $\mathbf{u}$ and $\mathbf{v}$ by $(\mathbf{u}, \mathbf{v}) = \mathbf{u}^T A \mathbf{v}$, where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Determine whether or not this bilinear form is positive definite.

X*. (Fall 2001) Give an example of a symmetric bilinear form on a real vector space which is not positive definite, and such that there is a basis $\{e_1, \ldots, e_n\}$ for the space with $(e_i, e_i) > 0$ for every i.