MATH 8253: ALGEBRAIC GEOMETRY
HOMEWORK 5
DUE WEDNESDAY, DECEMBER 14, 2:17 P.M.

INSTRUCTOR: SASHA VORONOV

Review your course notes and read Gathmann's *Chapter* 9: *Birational Maps and Blowing Up*, Sections 9.20–9.26, *Chapter* 10 : *Smooth Varieties*, and *Chapter* 12 : *Schemes*. Do Exercises 10.13, 10.17, 10.23, 12.14, 12.35, 12.42.

Hint to 10.23 (b): It is easier to show that the complement, the set of those f's (modulo scalars) for which $V_p(f)$ is not smooth is closed (and this set is not the whole $\mathbb{P}^{\binom{n+d}{n}-1}$). You should consider the *universal hypersurface* X, the set of points (f, x) in $\mathbb{P}^{\binom{n+d}{n}-1} \times \mathbb{P}^n$ such that $f(x) = 0$. From the projective Jacobi criterion, a point $x \in V_p(f)$ is not smooth, iff all the partials of f at x vanish. These equations define a closed subset Y of X, which is also closed. Study the image of Y under the projection, to $\mathbb{P}^{\binom{n+d}{n}-1}$.

Submit the homework by the start of our last, Wednesday, December 14, class meeting, *i.e.*, by 2:17 p.m. Please submit it electronically to Gradescope at
`https://www.gradescope.com/courses/445177`,
which you can access through Canvas.