MATH 8254: ALGEBRAIC GEOMETRY
PROBLEM SET 2, DUE MONDAY, MARCH 3, 2008

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From Hartshorne's textbook: II.5.11, II.7.8.

Problem 1.

(1) Let $S = \bigoplus_{n=0}^{\infty} S_n$ be a graded ring and $R = S_0$. Suppose $\phi : R \to A$ is a ring homomorphism and let $T = S \otimes_R A$ be the graded ring induced by the base change. Prove that $\operatorname{Proj} T \cong \operatorname{Proj} S \times_{\operatorname{Spec} R} \operatorname{Spec} A$.

(2) Let $\mathcal{S} = \bigoplus_{n=0}^{\infty} \mathcal{S}_n$ be a quasi-coherent graded $\mathcal{O}_X$-algebra over a scheme X. Let $f^*\mathcal{S} = \bigoplus_{n=0}^{\infty} f^*\mathcal{S}_n$ be the inverse image under a scheme morphism $f : Y \to X$. Prove that $\operatorname{Proj} f^*\mathcal{S} \cong \operatorname{Proj} \mathcal{S} \times_X Y$.

Problem 2. For a quasi-coherent sheaf $\mathcal{E}$ and an invertible sheaf $\mathcal{L}$ over a scheme X, prove that $\mathbb{P}(\mathcal{E})$ and $\mathbb{P}(\mathcal{E} \otimes \mathcal{L})$ are isomorphic schemes over X.

Problem 3. Let $f : Y \to X$ be a relative scheme. For an f-very ample sheaf $\mathcal{L}$ on Y, prove that for each integer $n > 0$, the sheaf $\mathcal{L}^{\otimes n}$ is also f-very ample.

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