

**MATH 8254: ALGEBRAIC GEOMETRY II**  
**HOMEWORK 3**  
**DUE FRIDAY, MARCH 3, 11:59 P.M.**

INSTRUCTOR: SASHA VORONOV

Review your favorite version of Nakayama's Lemma (e.g., Corollary 4.8 of Eisenbud) and Geometric Nakayama's Lemma [Vakil] 14.3.E (do not do the exercise, just take it for granted). Do **Exercises [Vakil]** 14.3.J, 14.3.K. (Note: we just called quasi-coherent sheaves of finite type *coherent*.)

Review your course notes, Gathmann's *Chapter 15: Differentials*, and study *Chapter 16: Cohomology of Sheaves*. Do **Exercises [Vakil]**: 22.2.T, 22.2.U, 19.3.A, 19.4.A.

*Hint. 22.2.T:* You may construct an  $\mathcal{O}_{X'}$ -module morphism by gluing it from a system of compatible module homomorphisms over a *distinguished affine base*, which consists of affine open subsets of  $X'$  and *distinguished inclusions*, the inclusions  $\text{Spec } A_f \rightarrow \text{Spec } A$  of a distinguished open subset into an affine open set. See Vakil 6.2.2(b) and 2.5.C. You may assume 6.2.2(c) and 2.5.1 which tell you that the sheaf of differentials, just as any  $\mathcal{O}_{X'}$ -module, may be given as a sheaf on the distinguished affine base. You may also assume that the restriction maps corresponding to distinguished inclusions for the sheaf of differentials are coming from localization, as this is the case for any quasi-coherent sheaf, see 6.2.D, which you may also take for granted.

*Correction. 22.2.U* (noticed by Connor): it should be  $p_1^*\Omega_{X/Z} \oplus p_2^*\Omega_{Y/Z}$ .

Submit the homework by the end of the due day, 11:59 p.m. Please submit it electronically to Gradescope at

<https://www.gradescope.com/courses/500650>,  
which you can access through Canvas.