Math 8272 **Homework 6**

Posted: 4/21/2016; Due: Friday, 4/29

The problem set is due at the beginning of the class on Friday, April 29, 2016.

Reading: Text: Sections 8.3–7.

Problems from Chapter 8 (Section 8.10): **2(2), 4, 5, 7** (The right-hand side of the formula for $K(\theta, \theta)$ must be changed to $1/h^\vee$; also, $\langle \rho, \theta^\vee \rangle := \rho(\theta^\vee)$, as always in the text).

Problem 1. Show that the weight lattice P is invariant under the Weyl group action.

Problem 2. Let P be a lattice in a real vector space $E \cong \mathbb{R}^r$ (*i.e.*, a subgroup of E which is isomorphic to $\mathbb{Z}^r$ and which spans E over the reals). Suppose that $W \subset \mathrm{GL}(E)$ is a finite subgroup preserving the lattice. Show there is a W-invariant symmetric bilinear form $(,)$ on E such that $(\lambda, \mu) \in \mathbb{Z}$ for all $\lambda, \mu \in P$. [*Remarks*: Of course, the notation is suggestive: in our context, $E = \mathfrak{h}^*_{\mathbb{R}}$, P is the weight lattice, and W is the Weyl group. However, the form $(,)$ will not be the one induced on $\mathfrak{h}^*_{\mathbb{R}}$ by the restriction of the Killing form to $\mathfrak{h}$, as we discussed in class on Monday, April 18. The form from this problem is used for defining the q-dimension in 8.38 and computing it for an irreducible representation L_λ.]