Math 8306 **Homework 1**

Posted: 9/8; Updated: 09/15; Due: Friday, 9/16/2016

The problem set is due at the beginning of the class on Friday, September 16.

Reading: Class notes. Text: Section 2.1 (pages 104-107, 110-113, 115-117). Beware that in the text these are done for singular homology, whereas we will be doing all that for simplicial homology.

Problem 1. Using the definition, compute directly the simplicial homology $H_\bullet(S^2; G)$ of the sphere S^2 represented as the boundary $\partial\Delta^3$ of the 3-simplex Δ^3. Here G is an abelian group (of coefficients for the homology).

Problem 2. Suppose that a simplicial complex K has n path components not intersecting with a subcomplex L. Show that $H_0(K, L; G) = nG$, the direct sum of n copies of G.

Problem 3. Suppose that a simplicial complex $K \cup L$ is the union of two subcomplexes K and L. Then, $K \cap L$, of course, is a simplicial complex, too. Show that $H_k(K, K \cap L; G) \cong H_k(K \cup L, L; G)$ for all k.

Problem 4. Given a commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \alpha} & & \downarrow{\scriptstyle \beta} & & \downarrow{\scriptstyle \gamma} & & \\ 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0 \end{array}$$

of abelian groups with exact rows, prove that there is an exact sequence

$$0 \to \operatorname{Ker} \alpha \to \operatorname{Ker} \beta \to \operatorname{Ker} \gamma \to \operatorname{Coker} \alpha \to \operatorname{Coker} \beta \to \operatorname{Coker} \gamma \to 0.$$

[*Hint*: Instead of doing it from scratch, you may use general procedures generating long exact sequences. In other words, you may use Theorem 2.16 from Hatcher.]

Problem 5. Thinking of the real projective plane $\mathbb{R}P^2$ as the sphere with antipodal points identified or, equivalently, the disk with antipodal points of its boundary identified, take an abstract simplicial complex (not just a Δ-complex!) (whose geometric realization is) homeomorphic to $\mathbb{R}P^2$. Do not prove that this is the case, but rather compute the homology $H_\bullet(\mathbb{R}P^2; \mathbb{Z})$ and $H_\bullet(\mathbb{R}P^2; \mathbb{Z}_2)$ by analyzing the simplicial chain complexes. [*Hint*: There should be quite a few simplices, we should have an example of a simplicial representation of $\mathbb{R}P^2$ in class with ten 2-simplices.]