**Math 8306** **Homework 5**

**Posted: 11/02; Updated: 11/07; Due: Friday, 11/11/2016**

The problem set is due at the beginning of the class on Friday, November 11.

**Reading**: Class notes. Text: Sections 2.1 (108–113, 119–126), 2.2 (149–153), 2.3 (160–162) and 3.1 (197–202).

**Conventions**: Homology by default means *singular homology.* No coefficients means integral coefficients: $H_n(X) := H_n(X; \mathbb{Z})$. A *closed manifold* is a smooth, compact manifold with no boundary.

**Problem 1.** Let $M$ be a closed orientable surface embedded in $\mathbb{R}^3$ in such a way that reflection across a plane $P$ defines a homeomorphism $r : M \to M$ fixing $M \cap P$, a collection of circles. Is it possible to homotope $r$ to have no fixed points? [*Hint*: Assess the situation using a problem from the previous homework. You may assume that tubular neighborhood of a circle fixed by $r$ is a cylinder.]

**Problem 2.** Show that $\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$ for all $n \geq -1$, where $SX$ is the suspension of $X$.

**Problem 3.** Prove that $H_n(X, Y) \cong H_n(X \cup CY, CY)$ for $n \geq 0$ and $H_n(X, Y) \cong H_n(X \cup CY)$ for $n \geq 1$.

**Problem 4.** Given a connected CW complex $X$ and a subcomplex $Y$, prove that $H_n(X, Y) \cong \tilde{H}_n(X/Y)$.

**Problem 5.** Show that $H_1(X, A)$ is not isomorphic to $\tilde{H}_1(X/A)$ if $X = [0, 1]$ and $A$ is the sequence $1, 1/2, 1/3, \dots$ together with its limit 0. [*Hint*: See Example 1.25 in Hatcher, Chapter 1.]

**Problem 6.** If $T_n(X, A)$ denotes the torsion subgroup of $H_n(X, A; \mathbb{Z})$, show that the functors $(X, A) \mapsto T_n(X, A)$, with the obvious induced homomorphisms $T_n(X, A) \to T_n(Y, B)$ and boundary maps $T_n(X, A) \to T_{n-1}(A)$, do not define a homology theory.

**Problem 7.** Show that the functors $H^n(X, A) = \mathrm{Hom}(H_n(X, A), \mathbb{Z})$ do not define a cohomology theory.

**Problem 8.** Let $p : X \to Y$ be a covering space with finite fibers of cardinality $n$. Using singular chains, construct a homomorphism $t : H_\bullet(Y; G) \to H_\bullet(X; G)$ such that the composite $p_* \circ t : H_\bullet(Y; G) \to H_\bullet(Y; G)$ is multiplication by $n$.