MATH 8307: ALGEBRAIC TOPOLOGY
PROBLEM SET 6, DUE MAY 9, 2003

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Problem 1. Show that if one of the differentials d' and d'' in a double complex vanishes, then the spectral sequence collapses at E^2.

Problem 2. Prove that if at least one of simplicial sets $K_\bullet$, $L_\bullet$ has finite type (i.e., finitely many nondegenerate elements in each degree), then $|K \times L|$ is homeomorphic to $|K| \times |L|$.

Problem 3. Show that if two simplicial maps $f, g : K \to L$ are homotopic ($f \sim g$, if there exists a simplicial map $H : K \times I \to L$, where $I = \Delta[1]$, such that $Hi_0 = f$ and $Hi_1 = g$), then $|f| \sim |g| : |K| \to |L|$. [Here $\Delta[1]$ is the simplicial set defined as the contravariant functor $\mathrm{Mor}_\Delta(-, [1])$ from the category Δ to the category of sets.]

Problem 4. Show that the bijections $\phi : SS(K_\bullet, \mathrm{Sing}_\bullet(X)) \to Top(|K|, X)$ and $\psi : Top(|K|, X) \to SS(K_\bullet, \mathrm{Sing}_\bullet(X))$ preserve homotopies.

Problem 5. Let G be a topological group, BG its classifying space, and ΩBG the based loop space of BG. Show that there exists a weak equivalence: $G \sim \Omega BG$.

Problem 6. Show explicitly, using the Milnor construction of a classifying space, that $B\mathbb{Z}_2 = \mathbb{RP}^\infty$ and $BS^1 = \mathbb{CP}^\infty$.

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