MATH 8307: ALGEBRAIC TOPOLOGY
PROBLEM SET 4, DUE FRIDAY, FEBRUARY 25, 2005

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From Hatcher's textbook:
Section 4.H: 3
Section 4.2: 30
Section 4.3: 8 ($ps = \mathrm{id}$ must be replaced with $\pi s = \mathrm{id}$), 9, 12, 17, 18

Problem 1. Show that any subspace of a weak Hausdorff space is weak Hausdorff. [Reminder: X is *weak Hausdorff*, if a continuous image of any compact (for us, this means compact and Hausdorff) space is closed in X.]

Problem 2. Show that any closed subspace of a k-space is a k-space. [Reminder: a space is called a *k-space*, if every compactly closed subspace is closed. A subspace $A \subset X$ is *compactly closed*, if the inverse image $g^{-1}(A)$ of A under any map $g : K \to X$ from a compact space K is closed.]

Problem 3. Show that an open subset U of a compactly generated space X is compactly generated, if each point in U has an open neighborhood in X with closure contained in U. [Reminder: a *compactly generated space* X is a weakly Hausdorff k-space. This is equivalent to saying that X is weakly Hausdorff and a subset $A \subset X$ is closed, if an only if the intersection of A with each compact subset of X is closed.]

Problem 4. Show that a cofibration (defined as a map $i : A \to X$ satisfying HEP) is an inclusion with closed image.

Problem 5. Let $i : A \to X$ be a cofibration, where A is contractible. Prove that the quotient map $X \to X/A$ is a homotopy equivalence.

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