MATH 8307: ALGEBRAIC TOPOLOGY
PROBLEM SET 6, DUE MONDAY, MAY 9, 2005

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From Hatcher's textbook:
Section 4.1: 11, 14, 15
Section 4.K: 3, 4
Section 4.2: 16

Problem 1. Give an example of two CW-complexes which have the same homology, but are not homotopy equivalent.

Problem 2. Suppose that X is a based space,

$$p : (CX, X) \to (CX/X, \{*\})$$

is the quotient map and

$$\partial : \pi_{q+1}(CX, X) \to \pi_q(X)$$

is the connecting homomorphism in the homotopy long exact sequence of the pair (CX, X). Prove that the following diagram commutes:

$$\begin{array}{ccc}
\pi_{q+1}(CX, X) & \xrightarrow{p_*} & \pi_{q+1}(CX/X) \\
\partial \big\downarrow & & \big\downarrow \cong \\
\pi_q(X) & \xrightarrow[\Sigma]{} & \pi_{q+1}(\Sigma X),
\end{array}$$

where Σ is defined (not from this diagram, as in class, but directly) as $\Sigma[\alpha] = [\Sigma\alpha]$, where $\alpha : S^q \to X$ represents a based homotopy class and $\Sigma\alpha : S^{q+1} \to \Sigma X$ its suspension. [Hint: remember we had $-\Sigma f$ in the cofiber sequence.]

Problem 3. Show that the two definitions of the Hurewicz homomorphism, one via applying homotopy to $X \to \mathrm{SP}\, X$ and the other via applying homology to $S^q \to X$, for a connected CW-complex X are the same.

Problem 4. Prove that the Whitney sum of two vector bundles $p : E \to B$ and $p' : E' \to B$ can be obtained as the bundle induced by the diagonal map $\Delta : B \to B \times B$, defined by $\Delta(x) := (x, x)$ for $x \in B$, from the product bundle $p \times p' : E \times E' \to B \times B$. In other words, show that

$$E \oplus E' \cong \Delta^*(E \times E').$$

Date: April 24, 2005.