

Answer for Midterm1

I. Definitions: Complete the following sentences

- 1: Odds[A] = $\frac{Pr[A]}{Pr[\text{not}A]}$
- 2: $LQ^A[B] = \frac{Pr[B|A]}{Pr[B|\text{not}A]}$
- 3: A polynomial of degree less than or equal to five, s.t. $f^{(n)} = P^{(n)}$ for $n \leq 5$
- 4: $\frac{1}{|A|} \int_A X(\omega) d\omega$
- 5: $LQ^A[B] = \frac{Pr[B|A]}{Pr[B|\text{not}A]}$

II. True or False

FTTFF

III. Computations

- 1:
 $E[XY] = 1 \times 0.1 + 0 \times 0.2 + 0 \times 0.5 = 0.3$
 $E[X] = 0.3$
 $E[Y] = 0$
 $\Rightarrow Cov[X, Y] = E[XY] - E[X]E[Y] = 0.3$

- 2:
a:
 $E[X|Y = 100] = 4.4$
 $E[X|Y = 200] = 2$
 $E[X|Y](\omega) = 4.4$ if $0 \leq \omega < 0.5$
 $= 2$ if $0.5 \leq \omega \leq 1$
b:
 $E[E[X|Y]] = 3.2$
 $SD[E[X|Y]] = \sqrt{Var[E[X|Y]]} = 1.2$

- 3:
 $(e^{8x} - e^8)_+$ is a function s.t it equals $(e^{8x} - e^8)$ if $e^{8x} - e^8 \geq 0$, that is $x \geq 1$, and is 0, otherwise.
 $\int_{-\infty}^{\infty} [(e^{8x} - e^8)_+] [e^{-\frac{x^2}{2}}] dx$
 $= \int_1^{\infty} [(e^{8x} - e^8)] [e^{-\frac{x^2}{2}}] dx$

$$\begin{aligned}
&= \int_1^\infty (e^{8x} e^{-\frac{x^2}{2}} dx - \int_1^\infty e^8 e^{-\frac{x^2}{2}} dx) dx \\
&= \int_1^\infty (e^{8x} e^{-\frac{x^2}{2}} dx - \int_1^\infty e^8 e^{-\frac{x^2}{2}} dx) dx \\
&= e^{32} \int_1^\infty (e^{-\frac{(x-8)^2}{2}} dx - e^8 \int_1^\infty e^{-\frac{x^2}{2}} dx) dx \\
&= e^{32} \int_{-7}^\infty (e^{-\frac{x^2}{2}} dx - e^8 \int_1^\infty e^{-\frac{x^2}{2}} dx) dx \\
&= \sqrt{2\pi}(e^{32}\phi(7) - e^8\phi(-1))
\end{aligned}$$

4:

$$\cos x = 1 - \frac{x^2}{2} + \text{higherorder} \dots$$

$$\Rightarrow \lim_{n \rightarrow \infty} [f(\frac{4}{\sqrt{n}})]^n = \lim_{n \rightarrow \infty} [1 - \frac{4^2}{2n}] = e^{-8}$$

5:

$$\text{Let } B = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}.$$

$$\Rightarrow B = \begin{pmatrix} 3 & 6 \\ 0 & 4 \end{pmatrix}$$

6:

$$Pr[B|A] = Pr[A|B] \frac{Pr[B]}{Pr[A]} = 0.32$$

7:

$$SD[A] = 0.2$$

$$Var[A] = 0.04$$

$$Var[X_1] = \frac{1}{100} Var[A]$$

$$SD[X_1] = \sqrt{Var[X_1]} = 0.02$$