hwa0014-0015

1 HW 0014: Row and column operations and linear algebra

0014-1:
step 1, transform into canonical form by row operations:

$$M = \begin{pmatrix} 1 & 1 & 1 & 5 & -22 \\ 8 & 8 & 9 & 18 & -179 \\ 3 & -2 & -2 & 5 & -11 \\ 2 & 6 & 7 & 19 & -22 \end{pmatrix}$$

$$\text{(Row operations)} \rightarrow N := \begin{pmatrix} 1 & 0 & 0 & 0 & -20 \\ 0 & 1 & 0 & 0 & -80 \\ 0 & 0 & 1 & 0 & 63 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

Let $\{v_1, v_2, v_3, v_4\}$ be row vectors of N. Because row operations don't change span of row vectors, the question becomes to determine if $v = (0, 3, 4, -5, -4)$ is in the span of $\{v_1, v_2, v_3, v_4\}$.
step 2, find coefficients:
suppose $v = a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4$. Plug in $v_1 = (1, 0, 0, 0, -20), v_2 = ...$
Comparing both sides, in order to make first four entries of both sides equal. a_1, a_2, a_3, a_4 should be $a_1 = 0, a_2 = 3, a_3 = 4, a_4 = -4$.
Check: If the 5th column is equal.
lefthand=-4≠0=$0 \times (-20) + 3 \times (-80) + 4 \times 63 + (-4) \times 3$=righthand.
step 3, conclusion based on whether we have find coefficients:
So, there are no value of a_1, a_2, a_3, a_4 satisfying $v = a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4$. Thus, v is not in the span$\{v_1, v_2, v_3, v_4\}$. Therefore, it is not in the span of original four vectors.

0014-2:

Let $v_1 = (1, 1, 1, 5); v_2 = (1, 2, 3, 4); v_3 = (0, 1, 2, -1); v_4 = (0, 0, 0, 0); v_5 = (1, 4, 8, 0); v_6 = (2, 5, 12, -1)$

- step 1 discard v_4

- step 2 check if $v_2 \in span\{v_1\}$ by method in 0014-1. v_1, v_2 are independent.
- step 3 check if $v_3 \in span\{v_1, v_2\}$ by method in 0014-1. $v_3 \in span\{v_1, v_2\}$, so discard v_3.
- step 4 check if $v_5 \in span\{v_1, v_2\}$. $v_5$is not in span$\{v_1, v_2\}$. So form new set $\{v_1, v_2, v_5\}$
- step 5 $v_6 \in span\{v_1, v_2, v_5\}$. Discard v_6

So, a basis of S is $\{v_1, v_2, v_5\}$.

0014-3:

In step 3 of 0014-2, we know $v_3 \in span\{v_1, v_2\}$.
So, $v_3 = a_1v_1 + a_2v_2$. substitution by $v_3 = ..., v_1 = ..., \Rightarrow (0, 1, 2, -1) = a_1(1, 1, 1, 5)+a_2(1, 2, 3, 4)$. Then, we have $a_1 = -1, a_2 = 1$, that is, $v_3 = -v_1+v_2$

0014-4:

Notation is the same as in 0014-1.

$$N := \begin{pmatrix} 1 & 0 & 0 & 0 & -20 \\ 0 & 1 & 0 & 0 & -80 \\ 0 & 0 & 1 & 0 & 63 \\ 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

Row operation will not change the kernel. So ker of M =ker of N.
Let $(a_1, a_2, a_3, a_4, a_5) \in$ ker N. Then

$$\begin{aligned} a_1 - 20a_5 = 0 \\ a_2 - 80a_5 = 0 \\ a_3 + 63a_5 = 0 \\ a_4 + 3a_5 = 0 \end{aligned}$$

Let $a_5 = 1$, we have (20,80,-63,-3,1) is in ker of M.
Next, determine the dimension of ker of M.
Transform N to full canonical form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

So, dim ker M= 1, that is, ker of M is generated by (20,80,-63,-3,1), i.e. ker M = span{(20,80,-63,-3,1)}

0014-5:

The fully canonical form is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

So, dim Im M=4.

0014-6:

a:

Let $M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & -5 & -11 \end{pmatrix}$ $M_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 0 & -2 & -10 \end{pmatrix}$

M (row and column operations)$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Because row operations don't change fully canonical form, M_1 has the same fully canonical form.

Then, dim ker M=dim ker M_1=1, dim Im M=dim Im M_1=2.

b:

Let $N = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & -5 & 8 \end{pmatrix}$ $N_1 = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ -2 & -5 & 3 \end{pmatrix}$

N (row and column operations)$\rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Because column operations don't change fully canonical form, N_1 has the same fully canonical form.

Then, dim ker N=dim ker N_1=0, dim Im N=dim Im N_1=3.

0014-7:

a:

A matrix is invertible if and only its kernel is zero if and only its fully canonical is identity matrix.

So, do row and column operation to find fully canonical form. the first matrix is not invertible, the second is invertible.

b:

To find inverse of the matrix$M = \begin{pmatrix} -1 & -1 & 1 \\ -2 & -3 & 2 \\ 1 & 3 & 0 \end{pmatrix}$: First, form a new matrix

$$\begin{pmatrix} -1 & -1 & 1 & 1 & 0 & 0 \\ -2 & -3 & 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Second, do row operations only for this new matrix. Transform the first three column into identity matrix. Then the last three column will be the inverse matrix.

inverse of M is $\begin{pmatrix} -6 & 3 & 1 \\ 2 & -1 & 0 \\ -3 & 2 & 1 \end{pmatrix}$

0014-8:

Write the equations in matrix form as follows:

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 3 \\ 3 & 8 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Let $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 3 \\ 3 & 8 & 4 \end{pmatrix}$

Then $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

By methods in 0014-7, we can find $A^{-1} = \begin{pmatrix} 0 & -4 & 3 \\ -1 & 2 & -1 \\ 2 & -1 & 0 \end{pmatrix}$

So $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & -4 & 3 \\ -1 & 2 & -1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

2 HW 0015: Determinants exist

0015-1:

$$\begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

0015-2:

-2

0015-3:

-2

0015-4:

$$\begin{pmatrix} 2 & 4 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

0015-5:

2

0015-6:

2

0015-7:

$$\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

0015-8:

0

0015-9:

0