hwa0023-0024

1 HW 0023: Principal component analysis

0023-1: **a**: Let R_1, R_2, R_3 be three rows of rotation matrix R.
Then $c_1[1,0,0]R = v_1 \Leftrightarrow c_1R_1 = v_1$. And we know that R_1 is a unit vector, so $c_1 = \sqrt{29}$, $R_1 = [2/\sqrt{29}, 3/\sqrt{29}, -4/\sqrt{29}]$.
Similarly, $c_2[0,1,0]R = v_2 \Leftrightarrow c_2R_2 = v_2$. And also, R_2 is a unit vector, so $c_2 = \sqrt{6}$, $R_2 = [1/\sqrt{6}, -2/\sqrt{6}, -1/\sqrt{6}]$.
Till now, we have found first two rows of R. The remaining task is to find another unit vector R_3 so that together with R_1, R_2, they form an orthonormal basis. Let $R_1, R_2, e_3 = [0,0,1]$ be a basis, and find R_3 by Gram-Schmit orthonormalization method as in section0019. $R_3 = [11/\sqrt{174}, 2/\sqrt{174}, 7/\sqrt{174}]$.
So, $R = \begin{pmatrix} 2/\sqrt{29} & 3/\sqrt{29} & -4/\sqrt{29} \\ 1/\sqrt{6} & -2/\sqrt{6} & -1/\sqrt{6} \\ 11/\sqrt{174} & 2/\sqrt{174} & 7/\sqrt{174} \end{pmatrix}$
Remark: v_1, v_2 happen to be orthogonal to each other. Otherwise, no such rotation matrix exists. **b**: By (a), $c_1e_1R = v_1$, so $v_1R^{-1} = c_1e_1 \in span\{e_1\}$. It is similar for v_2. So, $L = R^{-1} = R^t$

0023-2: Let M^t be the transpose of M. $MM^t = \begin{pmatrix} 7.75 & -(1.25)\sqrt{3} \\ -(1.25)\sqrt{3} & 5.25 \end{pmatrix}$
Then, we can find eigenvalues and eigenvectors of MM^t. As usual, Let $K = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$ be the rotation matrix such that $K(MM^t)K^t = diagonalmatrix$.
One may refer to previous sections to see how to find K.
$KM = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.8 & -2.4 \end{pmatrix}$.
Next, we need to find L so that $KML = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.8 & -2.4 \end{pmatrix} L$ is diagonal matrix $D = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \end{pmatrix}$.
Or, $DL^{-1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.8 & -2.4 \end{pmatrix}$
Let L_1, L_2, L_3 be three rows of L^{-1}, which are unit vectors. Then $aL_1 = [2,0,0], bL_2 = [0,1.8,-2.4]$. So, $L_1 = [1,0,0], L_2 = [0,0.6,-0.8]$. Since we have found first two rows of rotation matrix L^{-1}, It is routine to find L_3 by Gram-Schmit method.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.6 & 0.8 \\ 0 & -0.8 & 0.6 \end{pmatrix}$$

2 HW 0024: Cayleys Theorem

0024-1: **a**: $B = \begin{pmatrix} 8 & 12 & 1 \\ -10 & -15 & 13 \\ 7 & -18 & 8 \end{pmatrix}$

b: $B^t = \begin{pmatrix} 8 & -10 & 7 \\ 12 & -15 & -18 \\ 1 & 13 & 8 \end{pmatrix}$

c: $AB^t = \begin{pmatrix} 57 & 0 & 0 \\ 0 & 57 & 0 \\ 0 & 0 & 57 \end{pmatrix}$

d: $B^tA = \begin{pmatrix} 57 & 0 & 0 \\ 0 & 57 & 0 \\ 0 & 0 & 57 \end{pmatrix}$

e: $dI = \begin{pmatrix} 57 & 0 & 0 \\ 0 & 57 & 0 \\ 0 & 0 & 57 \end{pmatrix}$

0024-2

$$\begin{pmatrix} 2 & -7 & -3 \\ 4 & -3 & 6 \\ 9 & 5 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 1 \end{pmatrix}$$

$$z = \frac{\begin{pmatrix} 2 & -7 & 6 \\ 4 & -3 & 3 \\ 9 & 5 & 1 \end{pmatrix}}{\begin{pmatrix} 2 & -7 & -3 \\ 4 & -3 & 6 \\ 9 & 5 & -7 \end{pmatrix}}$$