

hwa0025

1 HW 0025: Basics of piecewise constant random variables

0025-1: a: $E(X) = 3 \times 0.4 + 9 \times 0.6 = 6.6 = E(Y)$

So, $E[X + Y] = E[X] + E[Y] = 13.2$

b: $E[X^2] = 9 \times 0.4 + 81 \times 0.6 = 52.2 = E[Y^2]$

$Var[X] = E[X^2] - (E[X])^2 = 52.2 - 6.6^2 = 8.64 = Var[Y]$

$E[XY] = (E[Y])^2 + \rho(Var[Y]) = 6.6^2 + \rho \times 8.64 = 43.56 + 8.64\rho$

$Cov[X, Y] = E[XY] - E[X]E[Y] = 43.56 + 8.64\rho - 6.6^2 = 8.64\rho$

$Var[X + Y] = Var[X] + Var[Y] + 2(Cov[X, Y]) = 2 \times 8.64 + 2 \times 8.64\rho = 17.28(1 + \rho)$

c: $\rho = -1$, $Var[X + Y] = 0$ is minimal.

d: $\rho = 1$, $Var[X + Y] = 34.56$ is maximal.