

hwa0026-0027

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(Dated: February 9, 2010)

0026-1:

$$Pr[A|B] = \frac{Pr[A\&B]}{Pr[B]} \rightarrow Pr[A\&B] = Pr[B]Pr[A|B] = 0.2 \times 0.9 = 0.18$$

$$Pr[B|A] = \frac{Pr[A\&B]}{Pr[A]} = \frac{0.18}{0.8} = 0.225$$

0026-2: a: There are infinite many pairs. one of such X,Y is as follows:

$$X(\omega) = 1, 0 < \omega < 0.8$$

$$X(\omega) = 0, 0.8 < \omega < 1$$

and

$$Y(\omega) = 0, 0 < \omega < 0.62$$

$$Y(\omega) = 2, 0.62 < \omega < 0.82$$

$$Y(\omega) = 0, 0.82 < \omega < 1$$

b: 0.225. No matter which pair of X,Y one chooses, the answer will be the same, following 0026-1.

0026-3: a: $\frac{Pr[A]}{Pr[notA]} = Odds[A] = 3 \rightarrow Pr[A] = 0.75, Pr[notA] = 0.25$

$$Pr[A\&B] = Pr[B|A]Pr[A] = 0.2 \times 0.75 = 0.15$$

$$Pr[notA\&B] = Pr[B|notA]Pr[notA] = 0.6 \times 0.25 = 0.15$$

$$Odds[A|B] = \frac{Pr[A|B]}{Pr[notA|B]} = \frac{Pr[A\&B]}{Pr[notA\&B]} = 1$$

$$\mathbf{b} : Pr[A\&B\&C] = Pr[C|B\&A]Pr[B\&A] = 0.8 \times 0.15 = 0.12$$

$$Pr[notA\&B\&C] = Pr[C|B\¬A]Pr[B\¬A] = 0.1 \times 0.15 = 0.015$$

$$Odds[A|B\&C] = \frac{Pr[A|B\&C]}{Pr[notA|B\&C]} = \frac{Pr[A\&B\&C]}{Pr[notA\&B\&C]} = 0.12/0.015 = 8$$

0027-1: $E[X] = 4 \times 0.65 + 10 \times 0.35 = 6.1$

$$Var[X] = E(X^2) - (E(X))^2 = 4^2 \times 0.65 + 10^2 \times 0.35 - 6.1^2 = 8.19$$

$$SD[X] = \sqrt{Var[X]} = \sqrt{8.19}$$

0027-2: a: $Pr[(Y=3)|(X=4)] = \frac{Pr[(Y=3)\&(X=4)]}{Pr[X=4]} = 0.3/0.65 = 6/13$

$$\mathbf{b} : E[Y|(X=4)] = 7 \times Pr[Y=7|X=4] + 3 \times Pr[Y=3|X=4] = 7 \times \frac{0.36}{0.65} + 3 \times \frac{6}{13} = 5.15$$