

Errata

The argument in question0029-2(c,d) does work for symmetric matrix. For general matrix, There is no such simple relation between positivity and eigenvalues.

Statement: M arbitrary matrix, C arbitrary invertible matrix, then: Positivity of M is the same as Positivity of C^tMC

Proof:

for a vector v, let $w = Cv$, then $v^tC^tMCv = [Cv]^tM[Cv] = w^tMw$. this will be (semi)positive if M is (semi)positive. Similarly, one can show that M is (semi)positive if C^tMC is.

This is not true for M and $C^{-1}MC$.

For general matrix, all we can say is the following

- (1) if there is one negative eigenvalues, the matrix will not be semipositive.
- (2) The fact that no eigenvalues less than 0 doesn't guarantee matrix be a semipositive.

The reason is

For (1): If $\lambda < 0$, v be the λ -eigenvector, then $v^tMv = v^t\lambda v = \lambda v^tv < 0$.

For (2): The difference between symmetric matrix and non-symmetric matrix is that:

(a) For symmetric matrix M, one can find orthonormal matrix A such that $A^{-1}MA = A^tMA = D(\text{diagonal})$.

(b) For non-symmetric M, one only have $AMA^{-1} = D$, and $A^{-1} \neq A^t$.

Therefore, by statement above, for symmetric matrix, the question of M being (semi)positive is reduced to the question whether D is (semi)positive, and is further linked to positivity of eigenvalues, because D is consist of eigenvalues as diagonal. While, For non-symmetric matrix, one can not always make the first reduction, and then, no similar conclusion on eigenvalues and positivity.

Back to the question0029-2, since CXC^{-1} is not symmetric, argument in c,d is not valid. So, there is no way but compute CXC^{-1} , and find $v=[1,0,0]$ will make $v^tCXC^{-1}v$ negative. Therefore, the answer will be **NO** in (c,d).