

hwa0038

Section0038: Multivariable change of variables

0038-1:

$$\text{Jacobi of } \psi = \frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \end{pmatrix} = r^2 \cos \phi. \text{ where } (x, y, z) = (r \cos \phi \cos \theta, r \cos \phi \sin \theta, r \sin \phi).$$

Change of variable:

(1) change domain: E to D;

(2) change variable: $dx \wedge dy \wedge dz = |Jac| \times dr \wedge d\theta \wedge d\phi$

(3) change function: 1 to 1

$$\text{Therefore, } \int_E 1 dx dy dz = \int_D 1 |Jac| dr d\theta d\phi = \int_D r^2 \cos \phi dr d\theta d\phi = \int_0^{r_0} r^2 dr \int_0^{\pi/2} \cos \phi d\phi \int_0^\pi d\theta = \frac{\pi r^3}{3}$$