hwa0039-0040

Section0039

0039-1:
a: Let $x = 2i + (2 + 4i - 2i)t$. $\int_{2i}^{2+4i} e^x dx = \int_0^1 e^{2i+(2+2i)t} d(2 + 2i)t = e^{2i+(2+2i)t}|_0^1 = e^{2+4i} - e^{2i}$
b: Similarly, the answer is $e^4 - e^{2+4i}$
c: The answer is $e^4 - e^{2i}$
d: Let $x = 1 + 2i + ht$. $e^{1+2i+h} - e^{1+2i} = \int_{1+2i}^{1+2i+h} e^x dx = \int_0^1 e^{1+2i+ht} d(ht) = he^{1+2i} \int_0^1 e^{ht} d(t) = he^{1+2i} e^{h\theta}$, where $0 < \theta < 1$, by Integral Mean Value Theorem.
Thus, $\lim_{h\to 0} \frac{e^{1+2i+h} - e^{1+2i}}{h} = \lim_{h\to 0} e^{1+2i} e^{h\theta} = e^{1+2i}$ by continuity of e^x and the fact that $e^0 = 1$.

0039-2:
a: Let $x = 2i + (2 + 4i - 2i)t$. $\int_{2i}^{2+4i} \bar{x} dx = \int_0^1 2i + (\bar{2} + 2i)td(2 + 2i)t = (2+2i) \int_0^1 -2i+(2-2i)tdt = (2+2i)(-2it+(2-2i)\frac{t^2}{2})|_0^1 = (2+2i)(-2i+(1-i)) = 8 - 4i$
b: Similarly, the answer is $-2 - 16i$
c: The answer is $6 - 8i$
d: Let $h = 1/j$. $\lim_{j\to\infty} \frac{1+2i\bar{+}1/j-1\bar{+}2i}{1/j} = 1$. Let $h = i/j$. $\lim_{j\to\infty} \frac{1+2i\bar{+}i/j-1\bar{+}2i}{i/j} = -1$. the limits are different when h approaches 0 by x-axis or y-axis respectively. This means the complex limit does not exist.
Remark: this method can only be used to prove some limit doesn't exist. It doesn't guarantee the existence of limit if two limits above coincide.

0039-3:
$V(\alpha(t)) = (-1 - 2t, 2), \alpha'(t) = (0, 2)$. So $V(\alpha(t)) \cdot \alpha'(t) = 4$. $\int_0^1 V(\alpha(t)) \cdot \alpha'(t)dt = \int_0^1 4dt = 4$
Similarly, $\int_0^1 V(\beta(t))\cdot\beta'(t)dt = 3$. $\int_0^1 V(\gamma(t))\cdot\gamma'(t)dt = -2$. $\int_0^1 V(\delta(t))\cdot\delta'(t)dt = -1$.
The answer is 4.

0039-4:
Let $D = [1, 2] \times [1, 3]$ be the area enclosed by K. $d\omega = 2dxdy$. By stokes' theorem, $\int_K \omega = \int_D d\omega = 2 \int_D dxdy = 4$

0039-5:
a: $\partial R = [1,2] \times 1 + 2 \times [1,3] + [2,1] \times 3 + 1 \times [3,1]$
b: $\int_{\partial R} \omega = \int_R d\omega$
c: $d\omega = 2dxdy$. $\int_R d\omega = 2\int_R dxdy = 2(2-1)(3-1) = 4$

0039-6:
For $f(z) = z^2$, $f_{\bar{z}} = \frac{\partial f}{\partial \bar{z}} = 0$. since $dz \wedge dz = 0$, $d\omega = (f_z dz + f_{\bar{z}} d\bar{z}) \wedge dz = f_z dz \wedge dz = 0$. $\int_K \omega = \int_D d\omega = \int_D 0 = 0$

0039-7:
If we identify real plane R^2 with complex plane C. Then, $z^2 = f(z) = P + iQ$, $dz = dx + idy$. So $\int_K (P+iQ)(dx+idy) = \int_K f dz = \int_K \omega = 0$

0039-8:
Let $x = 3+3t, y = 2+5t, z = 1+7t$. $\int_L xdx + ye^x dy - x^2 e^z dz = \int_0^1 3 + 3td(3+3t) + (2+5t)e^{3+3t}d(2+5t) - (3+3t)^2 e^{1+7t} d(1+7t) = ...$

0039-9:
$\int_R e^{2x-3y} dy \wedge dx = -\int_R e^{2x-3y} dx \wedge dy = -\int_8^9 \int_2^3 e^{2x-3y} dxdy = \frac{1}{6}(-e^{-18} + e^{-20} + e^{-21} - e^{-23})$

0039-10:
$d\omega = f_x dx + f_y dy + f_z dz + f_t dt$. $d(d\omega) = (f_{xx}dx + f_{xy}dy + f_{xz}dz + f_{xt}dt) \wedge dx + (f_{yx}dx + f_{yy}dy + f_{yz}dz + f_{yt}dt) \wedge dy + (f_{zx}dx + f_{zy}dy + f_{zz}dz + f_{zt}dt) \wedge dz + (f_{tx}dx + f_{ty}dy + f_{tz}dz + f_{tt}dt) \wedge dt = 0$, using the fact that $f_{xy} = f_{yx}, ...$

0039-11:
We need only to show that $d(d(Fdx \wedge dy)) = 0$.
$d(d(Fdx \wedge dy)) = d(F_z dxdydz + F_t dxdydt) = F_{zt} dtdxdydz + F_{tz} dzdxdydt = (F_{zt} - F_{tz}) dxdydtdz = 0$

0039-12:
a: Let $z = x + iy$. $e^{-5z} = e^{-5x-i5y} = e^{-5x} cos(-5y) + ie^{-5x} sin(-5y)$. $U = e^{-5x} cos(-5y), V = e^{-5x} sin(-5y)$
b: Yes.

0039-13:
a: $U = e^{3x^2-3y^2-2x+5} cos(6xy-2y), V = e^{3x^2-3y^2-2x+5} sin(6xy-2y)$
b: Yes.

0039-14:
a: $U = x^4 + y^4 + 2x^2y^2, V = 0$
b: No.

0039-15:

a: $U = x^4 + y^4 - 6x^2y^2, V = -4x^3y + 4xy^3$
b: No.

0039-16:
$f = 7z(2 + xy - 3y^2) - 3yz(5 - 2x^3)$
$g = (x^7 + 3)(2 + xy - 3y^2) - (5 - 2x^3)(x + 4y - z)$
$h = 3yz(x^7 + 3) - 7z(x + 4y - z)$

0039-17:
$f = 4x^3y^3(2x^2 + 3xyz) - 3y(x + 2xyz + 2z)$

0039-18:
Use the formula $df = f_x dx + f_y dy + f_z dz$.

0039-19:
$[csc(xyz) - xyzcot(xyz)csc(xyz)]dxdy - e^{yz}dxdz + [x^2ycsc(xyz)cot(xyz) - xze^{yz})]dydz$

Section0040

0040-1:
Notice that the direction of the wheel is parallel to the leg. $V(x, y) = (x/20, y/20)$.
So, the form $\omega = V(x, y) \cdot (dx, dy) = (xdx + ydy)/20$. $d\omega = 0$.
$\int_C \omega = \int_R d\omega = \int_R 0 = 0$.
It is not equal to the area of R. So, it is called mis-designed planimeter.