Math 1271 Quiz 2

31 January, 2012 Name: ________________________________

You have 15 minutes to take this quiz. No calculators, books, or notes are allowed. Show all of your work clearly. Good luck!

1. (*6 points*) Evaluate the limit, if it exists.

$$\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$$

Solution:

$$\begin{aligned}
&\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} \\
&= \lim_{t\to 0} \left(\frac{\sqrt{1+t}+\sqrt{1-t}}{t} \times \frac{(\sqrt{1+t}-\sqrt{1-t})}{(\sqrt{1+t}+\sqrt{1-t})} \right) \\
&= \lim_{t\to 0} \frac{(1+t)-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} \\
&= \lim_{t\to 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} \\
&= \lim_{t\to 0} \frac{2}{(\sqrt{1+t}+\sqrt{1-t})} \\
&= \frac{2}{1+1} \\
&= 1
\end{aligned}$$

2. (*6 points*) Find the numbers at which f is discontinuous. At which of these numbers is f from the right, from the left, or neither?

$$f(x) = \begin{cases} 1+x^2 & \text{if } x \le 0 \\ 2-x & \text{if } 0 < x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

Solution: Since $1+x^2$, $2-x$ and $(x-2)^2$ are polynomials, f is continuous on $(-\infty, 0)$, $(0, 2)$ and $(2, \infty)$. Now, consider the continuity of f at $x = 0$ and 2.

At $x = 0$,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (1+x^2) = 1$$

and

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2-x) = 2$$

So, f is discontinuous at 0. Since $f(0) = 1$, f is continuous from the left at 0.

At $x = 2$,

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2-x) = 0,$$
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-2)^2 = 0$$

and $f(2) = 0$. So, f is continuous at 2. The only number at whcih f is discontinuous is 0.