Calculus 1 - IVT, MVT, EVT, Fermat

Answer each question as either TRUE or FALSE

1) If a function f is defined and continuous for all (real) x and has a local minimum at $x = 2$, then $f'(2) = 0$.

FALSE - Derivative may not exist (for example, the absolute value function)

2) If a function f is defined and continuous for all (real) x and has an absolute minimum at $x = 2$, then $f'(2) = 0$.

FALSE - Derivative may not exist (for example, the absolute value function)

3) If a function f is defined on the interval $[a, b]$, then there is an input value c, with $a \leq c \leq b$ where f attains a local maximum.

FALSE - f may be discontinuous.

4) If a function f is defined on the interval $[a, b]$ and is continuous on that interval, then there is an input value c, with $a < c < b$ where f attains a local maximum value.

FALSE - f may be strictly increasing or decreasing, so that the extreme values happen at the endpoints.

5) If a function f is defined only on the interval $[a, b]$ and is differentiable on the interval (a, b) and is continuous at a and at b, then there is an input value c, with $a \leq c \leq b$ where f attains the maximum value.

TRUE - This is the Extreme Value Theorem. Note that differentiability on (a, b) implies continuity on (a, b).

6) If a function f is defined on the interval $[a, b]$ and $f(a) < f(b)$, then there is an input value c, with $a \leq c \leq b$ where $f'(c)$ is positive.

FALSE - Example, step function.

7) If a function f is defined on the interval $[a, b]$ and is continuous on that interval, and differentiable on (a, b), and $f(a) < f(b)$, then there is an input value c, with $a < c < b$ where $f'(c)$ is positive.

TRUE - This is the Mean Value Theorem

8) If $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$, then f is continous at zero.

FALSE - The limit may not be equal to $f(x)$, leaving a removable discontinuity.

9) If f is defined, continuous, and differentiable on the interval $[a, b]$ and f' is continous on the interval $[a, b]$, and if $f'(a) = 2$ and $f'(b) = 5$, then there is an input value c, with $a \leq c \leq b$ where $f'(c) = 3$.

TRUE - This is the Intermediate Value Theorem

10) If f is defined everywhere, and $f'(x)$ is never negative and $f'(x)$ is never positive, then f is constant.

FALSE - The derivative may not exist at some points, for example, step functions