Today's Homework Assignment

Section	Problems
10.3	3, 6, 11, 20, 36, 37, 44, 47, 53, 55
10.4	12, 17, 21, 23, 27, 36, 40

Outline

Sample Space

Definition

The set of all possible outcomes of a given experiment is called the **sample space.**

Examples
- If we flip a coin, the sample space can be written $S = \{H, T\}$.
- If we roll a die, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.
- If we flip two coins, the sample space can be written $S = \{(H, H), (H, T), (T, H), (T, T)\}$.

Our Canonical Example

Consider the experiment of rolling two fair six-sided dice. There are 36 possible outcomes:

$$S = \left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

Events

Definition

A subset of a sample space is called an **event.**

Examples
- In the experiment of flipping a coin, the event of a tail turning up is $E = \{T\}$.
- In the experiment of rolling a die, the event of an even number turning up is $E = \{2, 4, 6\}$.
- In the experiment of flipping two coins, the event of one head and one tail turning up is $E = \{(H, T), (T, H)\}$.

More Examples

Consider the experiment of rolling two fair six-sided dice. How many outcomes belong to each of the following events?

- A sum of 6 4 outcomes

$$\left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

- A 5 on exactly one die 10 outcomes

$$\left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

Sample Spaces and Events

Probability

Definition

If all outcomes in a sample space S are equally likely, then the **probability** of an event E is

$$P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ denotes the number of outcomes in E, and $n(S)$ denotes the number of outcomes in S.

Referring to the previous examples, if you roll two dice,

- the probability of getting a sum of 5 is $\frac{4}{36} = \frac{1}{9}$, and
- the probability of getting exactly one 5 is $\frac{10}{36} = \frac{5}{18}$.