Financial Mathematics 5001 : Homework 3 (0024 – 0025)

Due on 12 October 2011

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Solutions

0024 – 1. $A = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix}$, $N = \begin{pmatrix} 0 & 7 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 4 & 0 \\ 0 & 8 \end{pmatrix}$, $S = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$.

a) Compute AB and BA.
b) Compute ANB.
c) Determine if N is nilpotent.
d) Determine if ANB is nilpotent.
e) Compute ADB.
f) Determine if ADB is diagonal.
g) Compute ASB.
h) Determine if ASB is scalar.
i) Determine if D+N is a Jordan block.
j) Determine if S+N is a Jordan block.
k) Compute A(S+N)B.
l) Determine if A(S+N)B is a Jordan block.

a) $AB = BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 84 & -63 \\ 112 & -84 \end{pmatrix}$

c) $N^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, so N is nilpotent.

d) $(ANB)^2 = AN^2 B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, so ANB is nilpotent.

e) $\begin{pmatrix} -28 & 24 \\ -48 & 40 \end{pmatrix}$

f) ADB is not diagonal.

g) $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

h) ASB = 4 I, so it is scalar.

i) D is diagonal but not scalar and N is not standard nilpotent, so D + N is not a Jordan block.

j) S is scalar, but N is not standard nilpotent, so S + N is not a Jordan block.

k) $\begin{pmatrix} 88 & -63 \\ 112 & -80 \end{pmatrix}$

l) A (S + N) B has an entry on the subdiagonal, so it is not a Jordan block.

0024 – 2. Compute $\exp\begin{pmatrix} 6 & -2 \\ 0 & 6 \end{pmatrix}$.

$\begin{pmatrix} 6 & -2 \\ 0 & 6 \end{pmatrix}^i = \begin{pmatrix} 6^i & -2\,6^{i-1}\,i \\ 0 & 6^i \end{pmatrix}$. Therefore,

$$\sum \frac{\begin{pmatrix} 6 & -2 \\ 0 & 6 \end{pmatrix}^i}{i!} = \sum \frac{\begin{pmatrix} 6^i & -2\,6^{i-1}\,i \\ 0 & 6^i \end{pmatrix}}{i!} = \begin{pmatrix} e^6 & -2\,e^6 \\ 0 & e^6 \end{pmatrix}.$$

0024 – 3. Find a 2 x2 orthogonal matrix whose first row has entries –4/ 5, 3/ 5.

$\begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ c & d \end{pmatrix} . \begin{pmatrix} -\frac{4}{5} & c \\ \frac{3}{5} & d \end{pmatrix} = \begin{pmatrix} 1 & -\frac{4c}{5} + \frac{3d}{5} \\ -\frac{4c}{5} + \frac{3d}{5} & c^2 + d^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, so that $c = -3/5$ and $d = -4/5$ or $c = 3/5$ and $d = 4/5$.

So, $\begin{pmatrix} -4/5 & 3/5 \\ -3/5 & -4/5 \end{pmatrix}$ or $\begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix}$.

0024 – 4. Does there exist a 2 x2 orthogonal matrix whose first column has entries 2/ 3, 1/ 3?

No. $\begin{pmatrix} 2/3 & 1/3 \\ b & d \end{pmatrix} . \begin{pmatrix} 2/3 & b \\ 1/3 & d \end{pmatrix} = \begin{pmatrix} \frac{5}{9} & \frac{2b}{3} + \frac{d}{3} \\ \frac{2b}{3} + \frac{d}{3} & b^2 + d^2 \end{pmatrix}$, which cannot equal the identity because of the entry 5/ 9.

0025 – 1. Let $M = \begin{pmatrix} 1 & 1 & 4 & 11 & 6 \\ 2 & 1 & 7 & 16 & 8 \\ 5 & 4 & 19 & 49 & 26 \\ 7 & 1 & 22 & 41 & 18 \end{pmatrix}$.

a) Do "row magic" to M, indicating clearly all the row operations.

b) Put M in row canonical form, indicating clearly all the row operations.

c) Put M in fully canonical form, indicating clearly all the row and column operations.

d) Write $M = E_1 \ldots E_k\, C\, E_1' \ldots E_l'$ where $E_1 \ldots E_k$, $E_1' \ldots E_l'$ are elementary and C is fully canonical.

a) R2 = R2 – 2 R1 , R3 = R3 – 5 R1, R4 = R4 – 7 R1,

$$\begin{pmatrix} 1 & 1 & 4 & 11 & 6 \\ 0 & -1 & -1 & -6 & -4 \\ 0 & -1 & -1 & -6 & -4 \\ 0 & -6 & -6 & -36 & -24 \end{pmatrix}$$

R3 = R3 – R2, R4 = R4 – 6 R2

$$\begin{pmatrix} 1 & 1 & 4 & 11 & 6 \\ 0 & -1 & -1 & -6 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

b) Continuing from part a, R1 = R1 + R2, R2 = (–1) R2

$$\begin{pmatrix} 1 & 0 & 3 & 5 & 2 \\ 0 & 1 & 1 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

c) Continuing from part b, C3 = C3 − 3C1, C4 = C4 − 5C1, C5 = C5 − 2C1

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

C3 = C3 − C2, C4 = C4 − 6C2, C5 = C5 − 4 C2

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

d) Ri = Ri + kRj is given by the identity matrix with a k in row i, column j, multiplied on the left. Ci = Ci + kCj is given by the identity matrix with a k in row i, column j, multiplied on the right. Swapping rows or columns is done with the identity matrix where the 1 in row i column i and the 1 in row j column j now appear in row i column j and row j column i.

0025 −2. Solve

5w + 3x −2y −3 z = 3

−3 w + x −3 y −4 z = −18

4 w + x + 3 y + 4 z = 22

−3 w + 2 x + 2 y + 3 z = 6

We form the matrix

$$\begin{pmatrix} 5 & 3 & -2 & -3 & 3 \\ -3 & 1 & -3 & -4 & -18 \\ 4 & 1 & 3 & 4 & 22 \\ -3 & 2 & 2 & 3 & 6 \end{pmatrix}$$

and use elementary operations to obtain

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix}.$$

Therefore, w = 2, x = 1, y = −1, and z = 4.