

Financial Mathematics 5001: Homework 5 (0021-0022)

Due on 13 October 2010

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Solutions

0020-1

Compute

$$(2, -7, 4) \cdot (7, -4, -5).$$

$$14 + 28 + -20 = 22.$$

0020-2

Compute

$$(2, 4, 6, 8) \cdot (1, 2, 3, 4).$$

$$2 + 8 + 18 + 32 = 60.$$

0020-3Is $\{(2, 4), (4, 9)\}$ a linearly independent set in \mathbb{R}^2 ? Why or why not?

It is. Suppose that $a(2, 4) + b(4, 9) = (0, 0)$, with a, b not both 0. Then $(2a + 4b, 4a + 9b) = (0, 0)$, so that from the first coordinate we see that $2a + 4b = 0$ and $a = -2b$. But then using the second coordinate, we find that $4a + 9b = -8b + 9b = b = 0$, so that $a = b = 0$, which contradicts our assumption.

0020-4Is $\{(2, 4), (4, 9)\}$ a basis of \mathbb{R}^2 ? Why or why not?

It is. The dimension of \mathbb{R}^2 is two, so that the set—by the previous question—is a maximal set of linearly independent vectors. More explicitly, it is easy to construct the canonical basis $\{(1, 0), (0, 1)\}$ using this set.

0020-5Write $(2, 3, 4)$ as a linear combination of $(1, 2, 3)$, $(1, 2, 5)$, and $(1, 1, 0)$.

We want $a, b, c \in \mathbb{R}$ such that

$$(2, 3, 4) = a(1, 2, 3) + b(1, 2, 5) + c(1, 1, 0).$$

Solving this system of three linear equations in three unknowns gives $a = b = \frac{1}{2}, c = 1$.

0021-1Define $F : \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$F(x, y) = x^3 + 2x - 3y.$$

1. Compute $F(1, 2)$.
2. Compute $F(3, 5)$.
3. Compute $F(4, 7)$.
4. Is F linear? Why or why not?

1. $F(1, 2) = -3$.

2. $F(3, 5) = 18$.

3. $F(4, 7) = 51$.

4. It isn't.

$$F((1, 2) + (3, 5)) = F(4, 7) = 51 \neq 15 = -3 + 18 = F(1, 2) + F(3, 5).$$

0021-2

Let

$$M_1 := \begin{pmatrix} 6 & 0 & 1 & -9 & 1 \\ -7 & -3 & 4 & -5 & 0 \\ 0 & -1 & 0 & 2 & 1 \end{pmatrix}.$$

Compute $L_{M_1}(1, 2, 1, 2, 1)$.

$$L_{M_1}(1, 2, 1, 2, 1) = (-10, -19, 3).$$