Financial Mathematics 5001 : Homework 8 (0027 - 0028)

Due on 10 November 2010

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Solutions

0027 – 1. Write $\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix}$ as a product of elementary matrices.

We perform the following operations to write $\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix}$ in fully canonical form : $R1 = R1 * 1/2$, $R2 = R2 - 3\,R1$, $R1 = R1 - 2\,R2$. In matrices, we have that

$$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Multiplying by inverses, we obtain

$$\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

0027 – 2. Find $\det\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix}$.

$$\det\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix} = \det\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = 2 * 1 * 1 = 2.$$

0027 - 3. Find the signed area of the oriented parallelogram ((2, 3), (4, 7)).

From class, we know that the determinant gives us this signed area. Therefore, the answer is 2.

0027 – 4. Write $\begin{pmatrix} 2 & 4 & 0 \\ 3 & 7 & 0 \\ 0 & 0 & -4 \end{pmatrix}$ as a product of elementary matrices.

We can perform the same operations as in 0027 – 1 to get to the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$. Performing $R3 = R3 * -1/4$ gets us to fully canonical form. In matrices, we have that

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/4 \end{pmatrix}\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 0 \\ 3 & 7 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Multiplying by inverses, we obtain

$$\begin{pmatrix} 2 & 4 & 0 \\ 3 & 7 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix}.$$

0027 – 5. Find det $\begin{pmatrix} 2 & 4 & 0 \\ 3 & 7 & 9 \\ 0 & 0 & -4 \end{pmatrix}$.

$$\det\begin{pmatrix} 2 & 4 & 0 \\ 3 & 7 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \det\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix} = 2 * 1 * 1 * -4 = -8.$$

0027 - 6. Find the signed area of the oriented parallelogram ((2, 3, 0), (4, 7, 0), (0, 0, -4)).

From class, we know that the determinant gives us this signed area. Therefore, the answer is - 8.

0027 – 7. Write $\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ as a product of elementary matrices,

then a fully canonical matrix, then more elementary matrices.

We perform the following operations to write $\begin{pmatrix} 2 & 4 \\ 3 & 7 \end{pmatrix}$ in fully canonical form : R2 = R2 – 3 R1,

R2 = 1 / 7 R2, R3 = R3 – R2, R1 = R1 – R2, R1 = R1 * 1 / 2,
R3 = R3 * 1 / 2, R2 = R2 + R3, R1 = R1 – 2 R3. In matrices, we have that

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/7 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Multiplying by inverses, we obtain

$$\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

0027 – 8. Find det $\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & 1 \end{pmatrix}$.

$$\det\begin{pmatrix} 2 & 1 & 3 \\ 6 & 10 & 2 \\ 0 & 1 & 1 \end{pmatrix} = \det\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det$$
$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 * 7 * 1 * 1 * 2 * 2 * 1 * 1 = 28.$$

0027 - 9. Find the signed area of the oriented parallelogram ((2, 6, 0), (1, 10, 1), (3, 2, 1)).

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

From class, we know that the determinant gives us this signed area. Therefore, the answer is 28.

0028 – 1. Compute $\det\begin{pmatrix} -1 & 2 & -3 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix}$.

$$\det\begin{pmatrix} -1 & 2 & -3 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix} = -30.$$

(In anticipation of the next problem, do this by Laplacian determinant expansion by minors with the first row.)

0028 – 2. Compute $\det\begin{pmatrix} 2 & 4 & -8 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix}$.

$\det\begin{pmatrix} 2 & 4 & -8 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix} = -86.$ (Since only the first row has changed, we can reuse the minors from 0028 – 1.)

0028 – 3. Compute $\det\begin{pmatrix} -1+2 & 2+4 & -3-8 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix}$.

$$\det\begin{pmatrix} -1+2 & 2+4 & -3-8 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix} = \det\begin{pmatrix} -1 & 2 & -3 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix} + \det\begin{pmatrix} 2 & 4 & -8 \\ 4 & 5 & -9 \\ -7 & 8 & -9 \end{pmatrix} = -30 - 86 = -116.$$

0028 – 4. Compute $\det\begin{pmatrix} 1 & 9 & 4 & 3 & 0 \\ 0 & 2 & 1 & 4 & -1 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 0 & -3 & -4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.

The determinant of an uppertraianglular matrix is just the product of its diagonal entries. Therefore, the answer is - 6.
