Unassigned Problems

0066 - 1. Let N = 100.

a. Let $X \in \sum^{N} B^{0.25,u}_{0.75,d}$. Suppose E[X] = 1.03 and SD[X] = 0.24. Find d and u.

b. Let $Y \in \sum^{N} B^{p,u}_{1-p,d}$. Suppose d and u come from Part a . Choose $p \in (0, 1)$ such that $E[e^Y] = 1.02$. Find E[Y] and SD[Y].

a. We have that E[X] =
$N\left(E\left[B^{0.25,u}_{0.75,d}\right]\right) = 100\,(0.25\,u + 0.75\,d) = 1.03$ and that SD[X] = Sqrt[100 (0.25) (0.75)] (u − d) = 0.24. Solving,
we obtain that $u \approx 0.05187$ and $d \approx -0.003556$.

b. $E\left[e^Y\right] = E\left[\exp\left(\sum^{N} B^{p,u}_{1-p,d}\right)\right] = E\left[\prod^{N} \exp\left(B^{p,u}_{1-p,d}\right)\right] = E\left[\prod^{N} B^{p,e^u}_{1-p,e^d}\right] = E\left[B^{p,e^u}_{1-p,e^d}\right]^{100} = \left(p\,e^u + (1-p)\,e^d\right)^{100} = 1.02,$

so that using the values of u and d from part a, we have $p\,(1.05324) + (1-p)\,(0.99645) = 1.02^{(1/100)} = 1.0002$, so that $p \approx 0.066$.
Then $E[Y] = N\left(E\left[B^{0.066,u}_{0.934,d}\right]\right) =$
$100\,(0.066\,(0.05187) + 0.934\,(-0.003556)) \approx 0.01$ and SD[Y] = Sqrt[100 (0.066) (0.934)] (0.05187 + 0.003556) ≈ 0.138.

0066 – 1. Let $N = 10^9$.

a. Let $X \in \sum^{N} B^{0.25,u}_{0.75,d}$. Suppose E[X] = 1.03 and SD[X] = 0.24. Find d and u.

b. Let $Y \in \sum^{N} B^{p,u}_{1-p,d}$. Suppose d and u come from Part a . Choose $p \in (0, 1)$ such that $E[e^Y] = 1.02$. Find E[Y] and SD[Y].

a. We have that E[X] =
$N\left(E\left[B^{0.25,u}_{0.75,d}\right]\right) = 10^9\,(0.25\,u + 0.75\,d) = 1.03$ and that SD[X] = Sqrt$\left[10^9\,(0.25)\,(0.75)\right]$ (u − d) = 0.24. Solving,
we obtain that $u \approx 0.00001315$ and $d \approx -0.00000438$.

b. $E\left[e^Y\right] = E\left[\exp\left(\sum^{N} B^{p,u}_{1-p,d}\right)\right] = E\left[\prod^{N} \exp\left(B^{p,u}_{1-p,d}\right)\right] = E\left[\prod^{N} B^{p,e^u}_{1-p,e^d}\right] = E\left[B^{p,e^u}_{1-p,e^d}\right]^{10^9} = \left(p\,e^u + (1-p)\,e^d\right)^{10^9} = 1.02,$

so that using the values of u and d from part a, we have $p\,(1.00001) + (1-p)\,(0.999996) = 1.02^{(1/10^9)} = 1$, so that $p \approx 0.2857$.
Then $E[Y] = N\left(E\left[B^{0.2857,u}_{0.7143,d}\right]\right) = 10^9\,(0.2857\,(0.00001315) + 0.7143\,(-0.00000438)) \approx 628.321$ and SD[Y] =
Sqrt$\left[10^9\,(0.2857)\,(0.7143)\right]$ (0.00001315 + 0.00000438) ≈ 0.25.