MATH 2263 Spring 2012
Quiz 1 (Jan. 26, 2012)

Name _____ Student ID # _____ Section # _____

⋆ *Please show your work.*

1. A plane π is given by

$$3x - 2y + z + 4 = 0.$$

a. Please find a vector $\boldsymbol{n}$ that is orthogonal to the plane.
Answer: Any non-zero scalar multiple of the coefficient vector is OK. For example, we can choose $\boldsymbol{n} =< 3, -2, 4 >$.

b. A line l_1 is perpendicular to the plane and passes through the point $P(4, 5, 6)$. Please find parametric equations for l_1.
Answer: Since $l_1 \perp \pi$, the normal vector $\boldsymbol{n}$ in the 1st question is a direction vector of l_1. Now we have the two ingredients (a point and a direction) for the equations. From Equation 2 in Section 12.5, we have parametric equations:

$$x = 4 + 3t,\ y = 5 - 2t,\ z = 6 + t.$$

c. A line l_2 is given by

$$\frac{x - 20}{2} = y + 5 = \frac{z - 1}{-4}.$$

Are the two lines l_1 and l_2 perpendicular to each other? Why?
Answer: Two lines are perpendicular if and only if they have orthogonal direction vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. The latter is true if and only if the dot product is 0. From previous questions, we have a direction vector of l_1 is $\boldsymbol{v}_1 = \boldsymbol{n} =< 3, -2, 4 >$. The equations given here implies $\boldsymbol{v}_2 =< 2, 1, -4 >$. We have

$$\boldsymbol{v}_1 \cdot \boldsymbol{v}_2 = 3 \cdot 2 + (-2) \cdot 1 + 1 \cdot (-4) = 0.$$

Therefore, $\boldsymbol{v}_1 \perp \boldsymbol{v}_2$, and $l_1 \perp l_2$.

Remark: We can use boldface symbols for vectors, like $\boldsymbol{n}, \boldsymbol{v}$. When we write, we usually put an arrow on the symbols to distinguish from scalar, for example, $\vec{n}, \vec{v}$.