1. Problem 1.8(a), 1.21 in Holmes.

2. Consider the following eigenvalue problem:

\[-\frac{\partial^2 u}{\partial x^2} + \epsilon f(x) u = \lambda u, \quad \text{in} \quad \mathbb{R}/2\pi\mathbb{Z}, \tag{1}\]

where \(f\) is a smooth periodic function.

(a) Find all eigenvalues and eigenfunctions of the above problem when \(\epsilon = 0\).

(b) Discuss what happens with the eigenvalues and eigenfunctions when \(\epsilon \neq 0\) but small.

3. Consider the eigenvalue perturbation problem for the matrix \(A + \epsilon B\) where \(\epsilon > 0\) is a small parameter. Let \(\lambda_0\) be the eigenvalue from which we perform perturbations. Suppose the eigenvalue \(\lambda_0\) is an eigenvalue of geometric multiplicity 1 but of algebraic multiplicity 3 (that is to say, eigenspace is one-dimensional but the generalized eigenspace is 3-dimensional). Find the leading order correction to the eigenvalue and eigenvector.

4. Consider the following problem in two spatial dimension, where \(r\) and \(\theta\) are the polar coordinates:

\[
\Delta \phi = 0 \quad \text{for} \quad r > a(1 + \epsilon \cos(2\theta)), \\
\phi = 0 \quad \text{at} \quad r = a(1 + \epsilon \cos(2\theta)) \\
\phi = r \cos \theta \quad \text{as} \quad r \to \infty. \tag{2}\]

Find the solution to the above problem to order \(\epsilon\).